UNIVERSITY OF ILLINOIS
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Agricultural Supply Chain Design Considering Post-Harvest Loss, Congestion, Land Use Competition, and Infrastructure Investment

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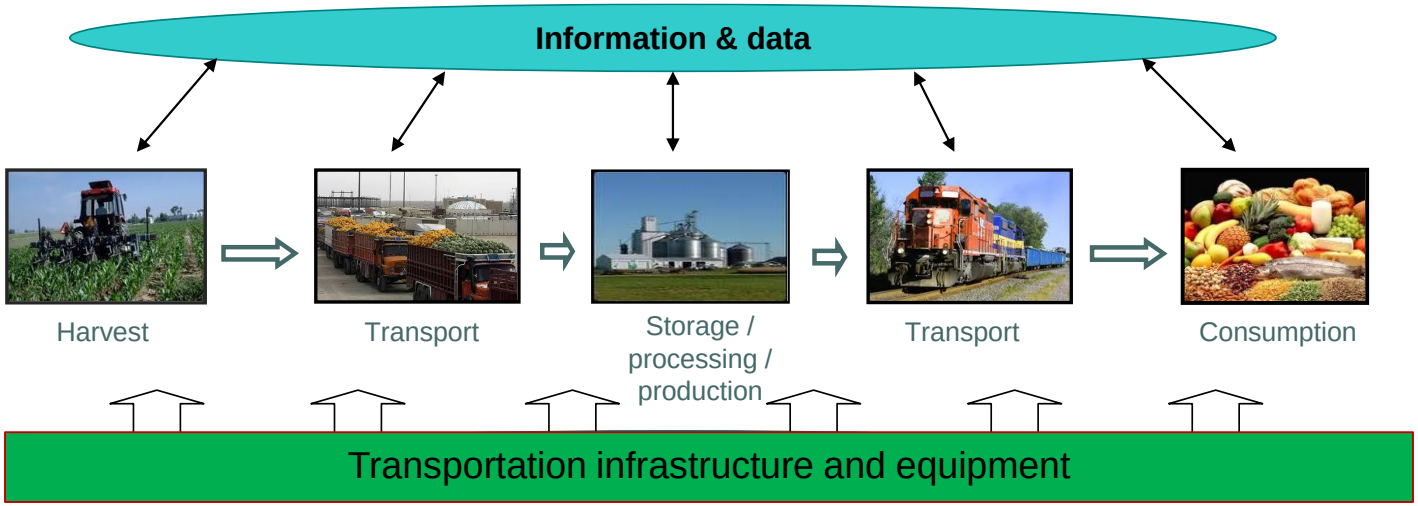
International Seminar on Agroindustrial Logistics
ESALQ-LOG, April 10, 2017

Agricultural Supply Chain & Logistics

- Integrated, Efficient, and Resilient Logistics Systems



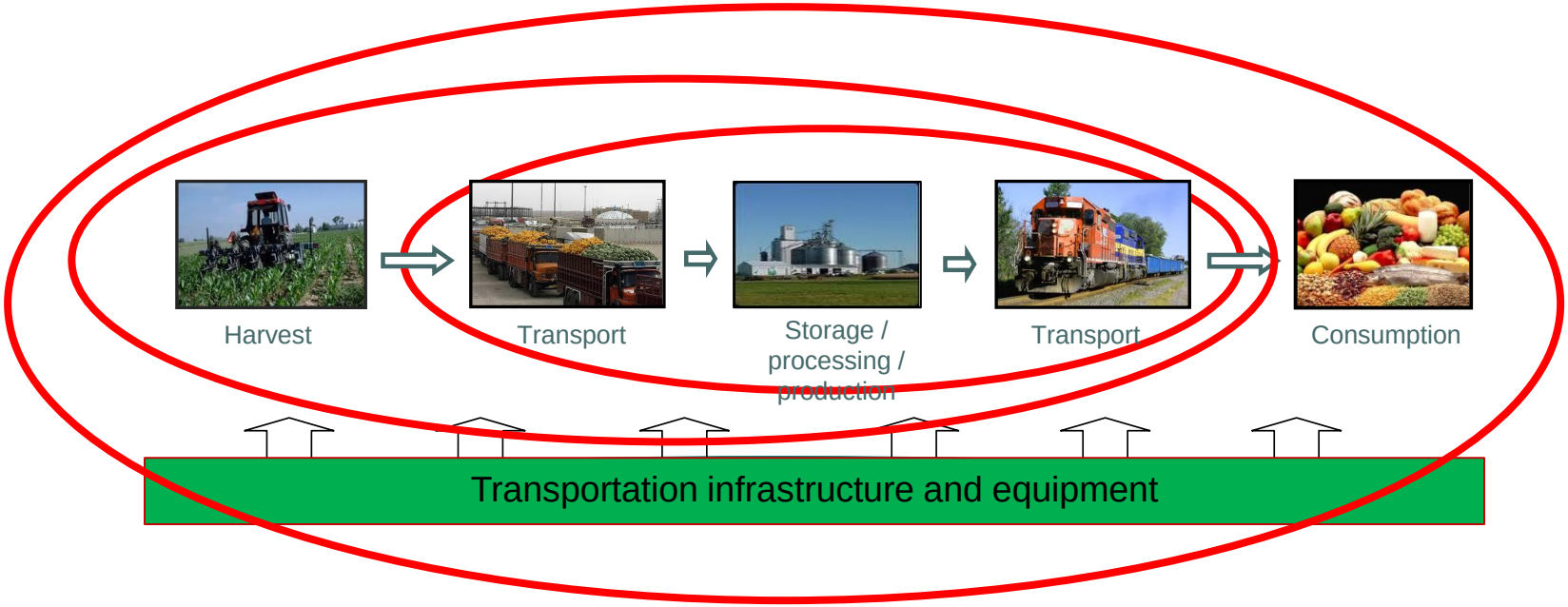
Production Strategy
- Supplier evaluation and selection
- Contracting and sourcing
- Pricing and competition

Location Strategy
- Number/size/location of facilities
- Assignment of stocking to sourcing
- Assignment of demand to stocking or sourcing

Transport Strategy
- Modes of transport
- Carrier routing and scheduling
- Shipment size and consolidation

Inventory Strategy
- Inventory levels
- Deployment of inventories
- Replenishment and control

Today's Topics



1. Agricultural supply chain design considering post-harvest loss and shipment congestion
2. Agricultural land use competition and government regulations
3. Joint planning of agricultural supply chain and roadway infrastructure rehabilitation

Agricultural supply chain design considering post-harvest loss and shipment congestion

Post harvest loss (PHL)

- Degradation in both quantity and quality loss of grains during transportation, processing and storage
  - Quantity loss: spillage, drying
  - Quality loss: Fungi, broken and cracked grains

- 1.3 billion tons of food wasted/lost worldwide each year
  - Up to 20% of total production in developing countries

Physical losses in traditional postharvest chain

Cutting, handling
1-5%

Manual threshing
1-5%

Sun drying
3-5%

Open storage
5-10%

Village milling
20-30%

Small retailers

Quality losses resulting in 10-30% loss in value

Machine threshing
1-5%

Combine harvesting
1-5%

Mechanical drying
1-2%

Sealed storage
1-2%

Commercial milling
5-30%

Large retailers

Physical losses in mechanized postharvest chain

Storage deficiency

- Brazil: current storage capacity at 75% of grain production vs. ideal target of 120% (FAO)
  - Ship grains during harvest season
  - Congestion at export ports and elevators → high PHL
  - High logistic cost, 20% higher

40-mile lineup of trucks (around 2 weeks) waiting to unload soybeans/corns outside Santos Ports and various rail terminals at Araguaia, Brazil, 2013

Objective

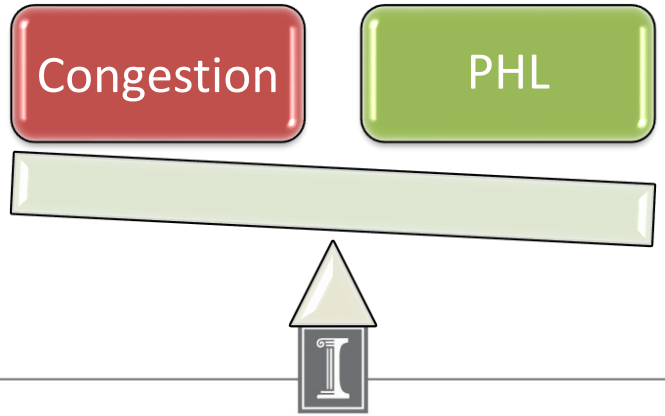
- Brazilian government has a five-year-program of Storage Construction (R$ 5 billion low-interest loans per year from 2013 to 2018)
- Strategic grain processing/storage facility location problem considering stochastic crop yield to reduce PHL
  - Food company-two strategies: risk-averse vs. risk-seeking
  - Farmers: non-cooperative

bi-level Stackelberg leader-followers game
& robust optimization

Harvesting timing

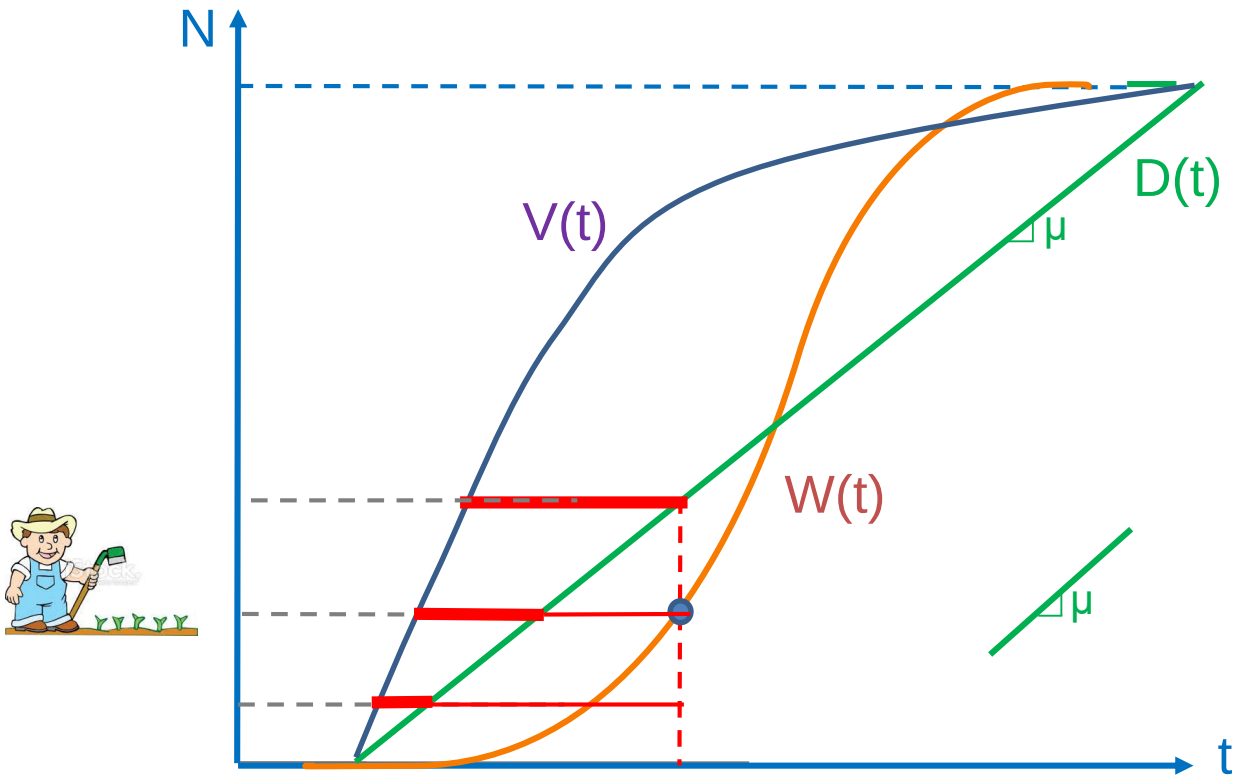
- Determined by the degree of maturity
  - Wished time: excessive waiting time at ports due to their limited service rate to load grains to ships
  - Early harvest: weight reduction, high moisture content
  - Late harvest: quantity losses caused by rodents and insects

A story about farmers in Brazil having no choice but to let soybean rot in the fields owing to bad harvest timing decisions



The Time Dimension

- Given a facility with capacity μ
- Serving farmers with a wished harvesting schedule W(t)
- The farmers choose their actual harvesting times knowing that congestion may occur at the facility
- Their collective decisions will determine the V(t) and D(t) curves – the actual cumulative harvesting curve and the actual "processing" time
- How would they each make the 'best' decision?



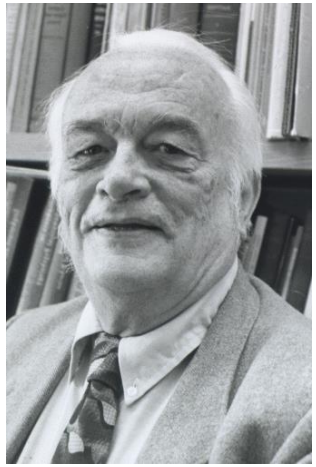
<u>Wish Curve, *W(t)*</u>: the cumulative amount of farmers who wish to harvest by time *t*.

Vickrey Equilibrium

- Suppose that each farmer values
  - time in queue at a cost rate β (\$/day)
  - early harvest penalty at cost rate $e\beta$ (\$/day)
  - late harvest penalty at cost rate $L\beta$ (\$/day)

- **<u>Vickrey's Equilibrium Principle</u>**: no farmer should be able to decrease its generalized cost by changing their harvest time.

- Under equilibrium
  - farmers harvest in the same sequence as their wish curve
  - equilibrium harvest curve $V(t)$ is piecewise linear



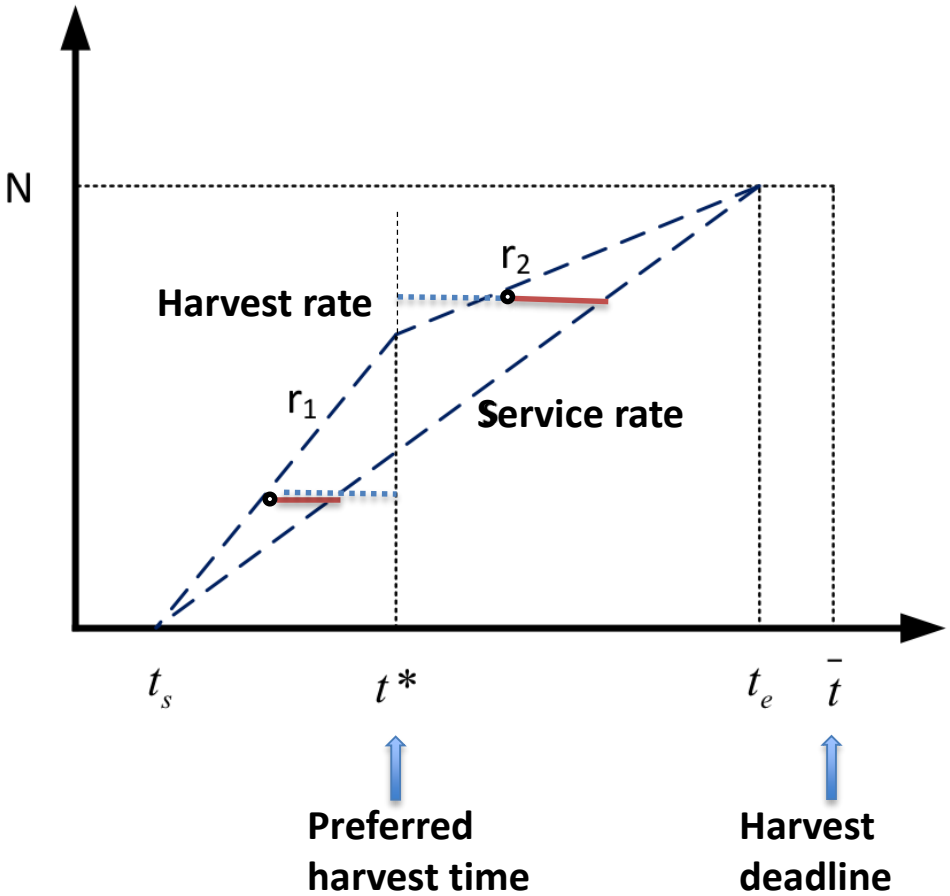
William Vickrey (1914-1996)
Columbia U
1996 Nobel Laureate Econ

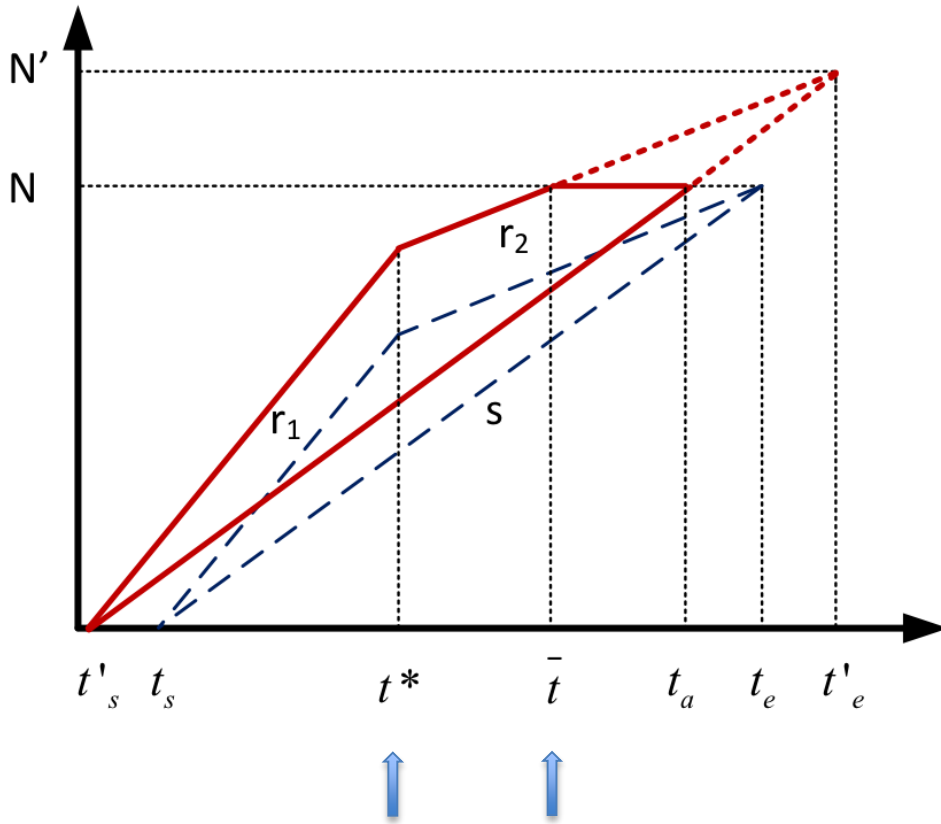
John F. Nash (1928-2015)
Princeton U
1994 Nobel Laureate Econ

Harvesting timing equilibrium

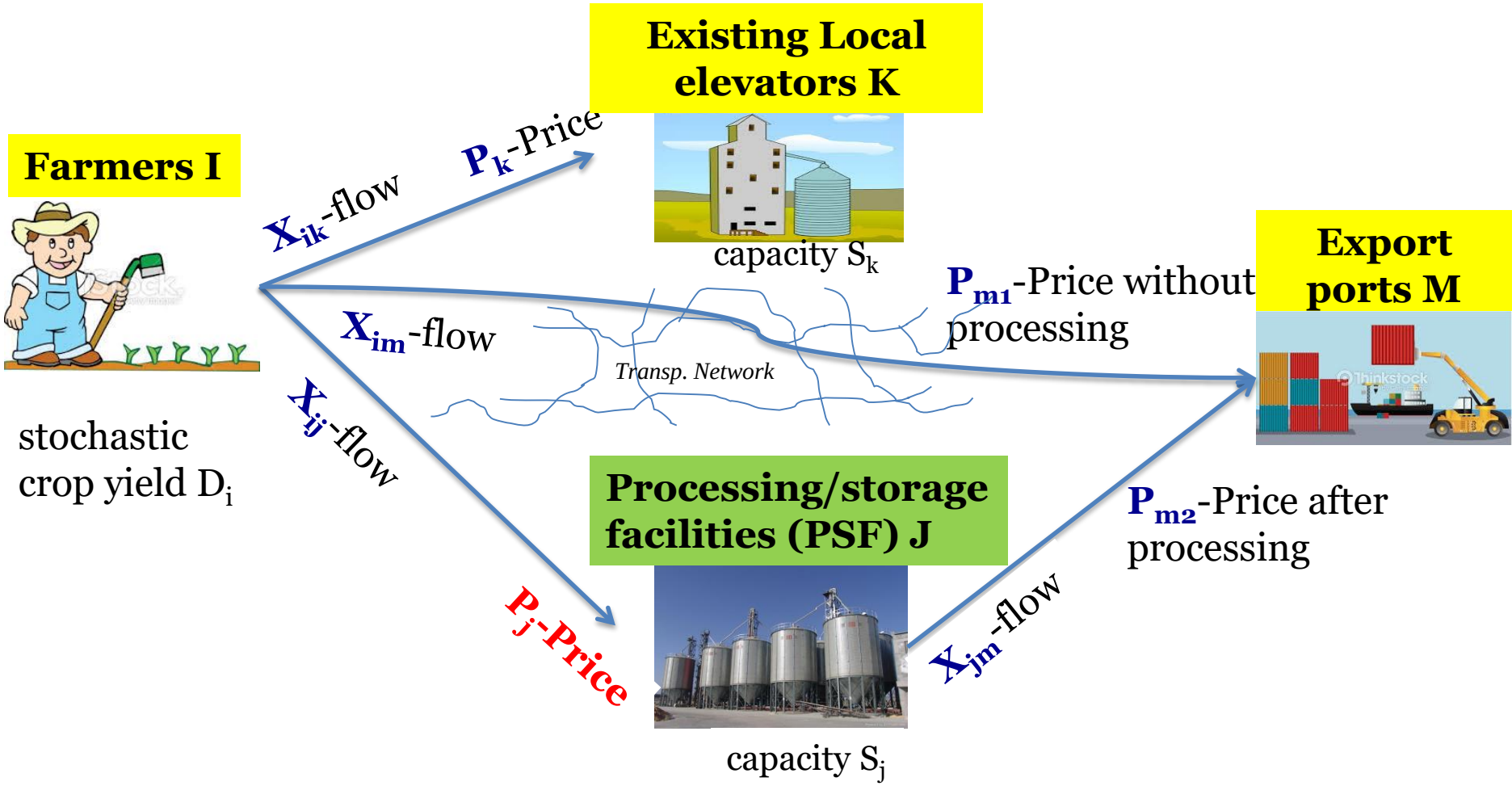
- Generalized congestion cost =

queuing cost + early harvest penalty + late harvest penalty



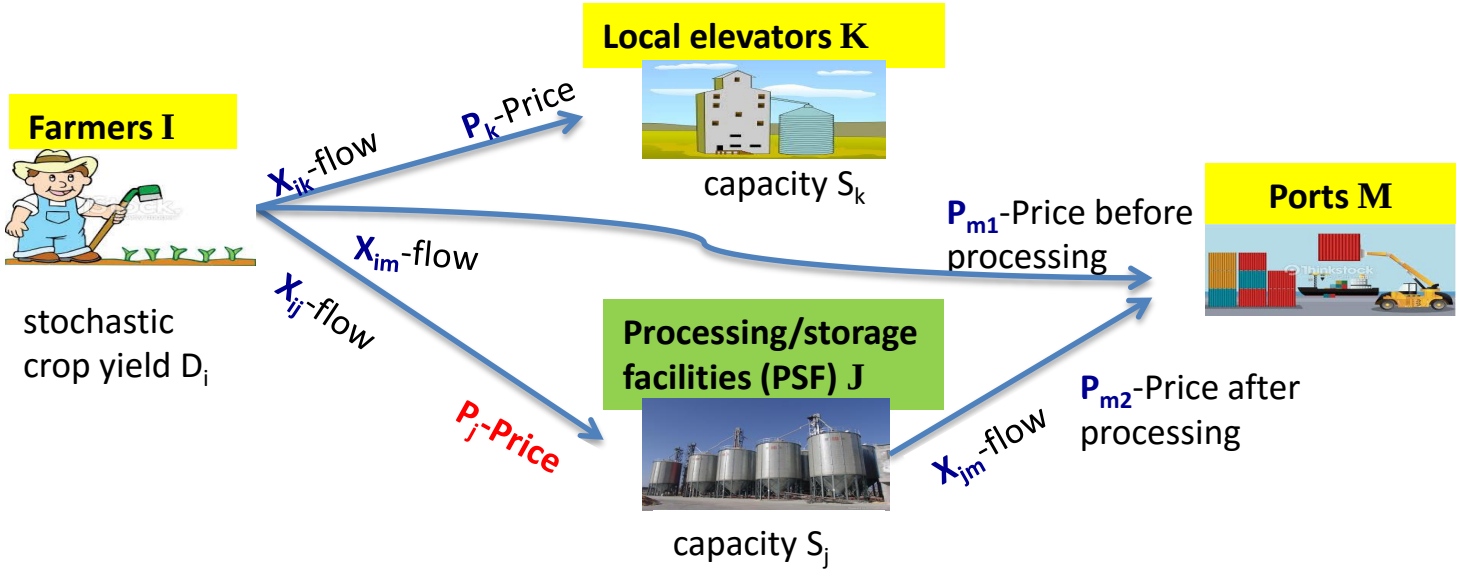


Three-echelon supply chain network



- Farmers: transportation, PHL cost from farmlands to the point of sales
- Food Company: grain processing and shipment cost from PSFs to ports

Three-echelon supply chain network



- Decision variables
  - Food Company: PSF location Y_j , grain purchase price P_j, grain flow X_{jm}
  - Farmers: grain flow X_{ik}, X_{im}, X_{ij}
- Parameters
  - Purchase price: P_k, P_{m1}, P_{m2} --linearly decrease with the grain throughput
  - PHL rate before & after processing, transportation cost, etc.
  - **Port/PSF congestion cost from Vickrey model.**

Risk-averse food company

- Privately owned, profit maximization
- Worst-case scenario (lowest crop yield)

Decision variables

$\mathbf{Y}, P_J$

(**P1-1**) Upper level:

$$\underset{\mathbf{x_{JM}},\mathbf{Y},\mathbf{p_J}}{Max} \sum_{j\in J}\sum_{m\in M} x_{jm}p_{m2}\left(1-\hat{c}_4 l_{jm}-\tilde{c}_4\right)-\sum_{j\in J}\sum_{m\in M} x_{jm}\left(p_j+c_3 l_{jm}+c_2^j\right)-\sum_{j\in J} c_1^j Y_j \quad (1)$$

Revenue - procurement - processing - PHL - transportation -PSF construction

$$\text{s.t.}\quad Y_j \in \{0,1\}\ \forall j\in J \quad (2)$$

Integrality constraints

$$\sum_{m\in M} x_{jm} = \sum_{i\in I} x_{ij}\left(1-c_4 l_{ij}\right)\quad \forall j\in J \quad (3)$$

Grain flow conservation

where x_{ij}, $j\in J$, solves (4)-(7) below for each farmer $i\in I$,

(**P1-2**) Lower level:

X_{IJ}, X_{IK}, X_{IM}

$$\underset{\mathbf{x_{IJ}},\mathbf{x_{IK}},\mathbf{x_{IM}}}{Max} \sum_{k\in K}\left[p_k\left(1-c_4 l_{ik}\right)-c_3 l_{ik}\right]x_{ik}+\sum_{j\in J}\left[p_j\left(1-c_4 l_{ij}\right)-c_3 l_{ij}\right]x_{ij} + \sum_{m\in M}\left[p_{m1}\left(1-c_4 l_{im}\right)-c_3 l_{im}-c_0^m\right]x_{im}-c_5\bar{d}_i \quad (4)$$

Sales - production - PHL - transportation - port congestion

$$\text{s.t.}\quad \sum_{k\in K} x_{ik}+\sum_{m\in M} x_{im}+\sum_{j\in J} x_{ij} \le D_i,\ \forall \mathbf{D}\in U \quad (5)$$

Grain flow conservation

$$\sum_{i\in I} x_{ik} \le S_k \quad \forall k\in K \quad \leftarrow \mu_k \quad (6)$$

Capacity constraints at elevators

$$\sum_{i\in I} x_{ij} \le S_j Y_j \quad \forall j\in J \quad \leftarrow \rho_j \quad (7)$$

Capacity constraints at PSF

Stochastic crop yield

$$\sum_{k \in K} x_{ik} + \sum_{m \in M} x_{im} + \sum_{j \in J} x_{ij} \le D_i, \forall \mathbf{D} \in U$$

- crop yield of farmland D_i," i Î I , fluctuate within a predetermined uncertainty set

$$U \equiv \left\{ \mathbf{D} : D_i \in \left[\overline{d}_i - \hat{d}_i, \overline{d}_i + \hat{d}_i \right], \forall i, \sum_{i \in I} \frac{\left| D_i - \overline{d}_i \right|}{\hat{d}_i} \le \Gamma \right\}$$

$\overline{d}_i$: nominal value of crop yield

$\hat{d}_i$: half length

G : deviation budget

Uncertainty constraints linearization

- Polyhedral uncertainty set

$$U \equiv \left\{ \mathbf{D} : D_i \in \left[\overline{d}_i - \hat{d}_i, \overline{d}_i + \hat{d}_i \right], \forall i, \sum_{i \in I} \frac{\left| D_i - \overline{d}_i \right|}{\hat{d}_i} \le \Gamma \right\}$$

Primal: $\min \ \overline{d}_i + Z_i \hat{d}_i \ \ s.t. \ \sum_{i \in I} |Z_i| \le \Gamma, \ |Z_i| \le 1$ $\Leftrightarrow$ **Dual:** $\max \ \overline{d}_i - q\Gamma - r_i \ \ s.t. \ q + r_i \ge \hat{d}_i, \ q \ge 0, r_i \ge 0$

Rewrite the uncertainty related constraints (5) by deterministic linear const., removing the uncertainty parameters

$$\sum_{k \in K} x_{ik} + \sum_{m \in M} x_{im} + \sum_{j \in J} x_{ij} \le \left(\overline{d}_i - q\Gamma - r_i \right) \quad \leftarrow \lambda_i$$

$$q + r_i \ge \hat{d}_i \quad \leftarrow \theta_i$$

$$q \ge 0, r_i \ge 0$$

KKT conditions for the lower level problem

- discretely constrained mathematical problem with equilibrium constraints (DC-MPEC)

$$0 \le x_{ik} \perp -p_k\left(1-c_4 l_{ik}\right)+c_3 d_{ik}-a_k\left(1-c_4 l_{ik}\right)x_{ik}+\lambda_i+\mu_k \ge 0 \quad \forall\, i \in I, k \in K \tag{1}$$

$$0 \le x_{ij} \perp -p_j\left(1-c_4 l_{ij}\right)+c_3 l_{ij}+\lambda_i+\rho_j \ge 0 \quad \forall\, i \in I, j \in J \tag{2}$$

$$0 \le x_{im} \perp -p_{m1}\left(1-c_4 l_{im}\right)+c_3 l_{im}+c_0^m-a_{m1}\left(1-c_4 l_{im}\right)x_{im}+\frac{\partial c_0^m}{\partial x_{im}}x_{im}+\lambda_i \ge 0, i \in I, m \in M \tag{3}$$

$$0 \le \lambda_i \perp \overline{d}_i-\gamma G-r_i-\sum_{k \in K} x_{ik}-\sum_{m \in M} x_{im}-\sum_{j \in J} x_{ij} \ge 0 \quad \forall\, i \in I \tag{4}$$

$$0 \le \mu_k \perp S_k-\sum_{i \in I} x_{ik} \ge 0 \quad \forall\, k \in K \tag{5}$$

$$0 \le \rho_j \perp S_j Y_j-\sum_{i \in I} x_{ij} \ge 0 \quad \forall\, j \in J \tag{6}$$

- Lagrangian relaxation based algorithm

Risk-seeking food company

- State-owned, consider both its profit and social responsibility (e.g., providing sufficient storage space)
- Based on a bumper year scenario

for each farmer $i \in I$,

(**P4-2**) Lower level:

$$\underset{\mathbf{x_{IJ}},\mathbf{x_{IK}},\mathbf{x_{IM}}}{Max} \sum_{k\in K}\left[p_k\left(1-c_4 l_{ik}\right)-c_3 l_{ik}\right]x_{ik}+\sum_{j\in J}\left[p_j\left(1-c_4 l_{ij}\right)-c_3 l_{ij}\right]x_{ij}$$
$$+\sum_{m\in M}\left[p_{m1}\left(1-c_4 l_{im}\right)-c_3 l_{im}-c_0^m\right]x_{im}-c_5\overline{d}_i$$

s.t. (6), (7) and

$$\sum_{k\in K} x_{ik}+\sum_{m\in M} x_{im}+\sum_{j\in J} x_{ij}\le d_i$$

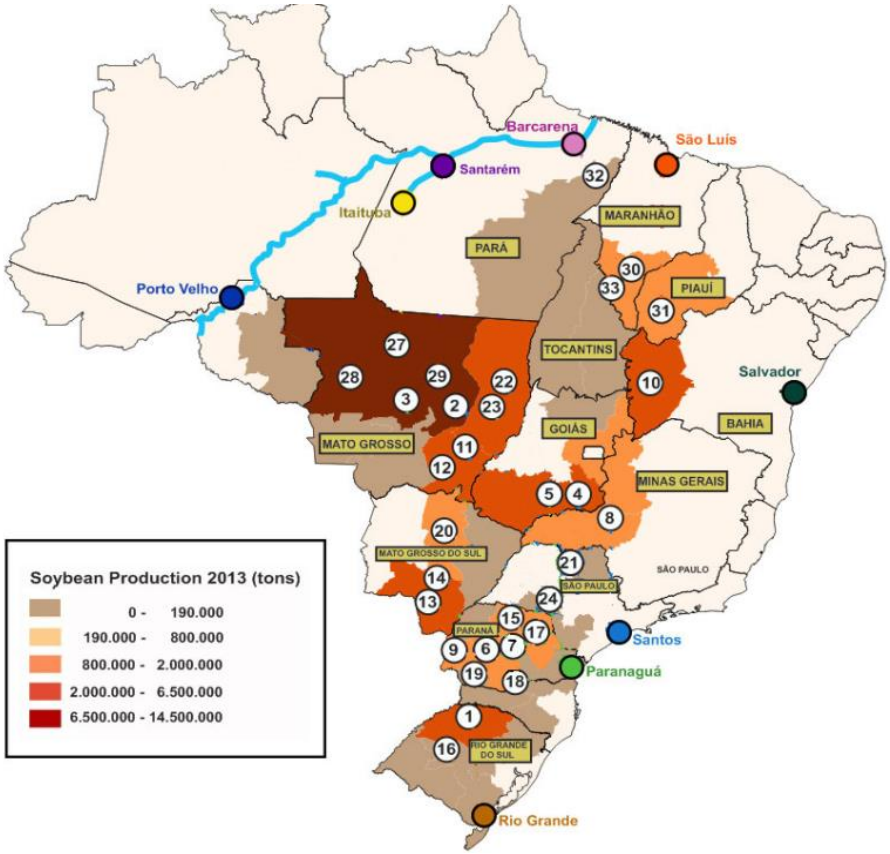
$$\overline{d}_i-\hat{d}_i\le d_i\le \overline{d}_i+\hat{d}_i \quad \forall i\in I;\quad \sum_{i\in I}\frac{\left|d_i-\overline{d}_i\right|}{\hat{d}_i}\le \Gamma$$

Grain flow conservation

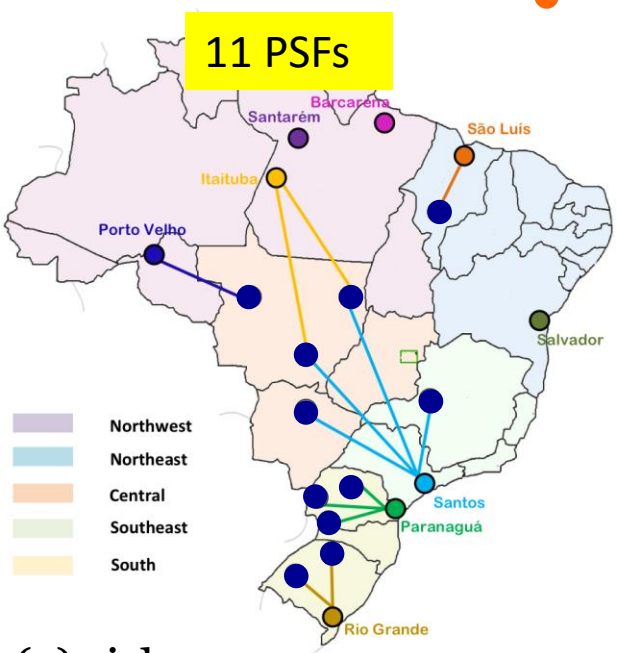
Brazil case study

- **33** major agricultural regions (83% of Brazil's total soybean production)
- **18** existing elevators (center of states with the highest storage capacity)
- **9** largest ports

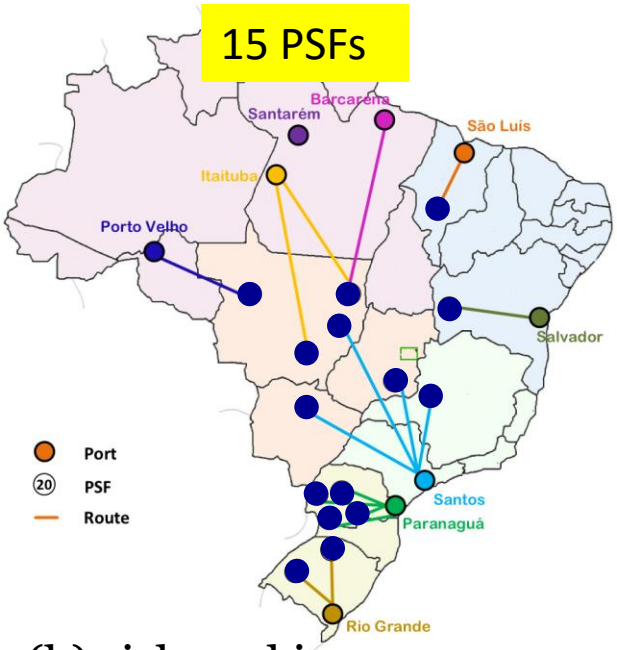
  (90% of total soybean exports)
- maximum deviation of the stochastic soybean production is **19%**
- deviation budget is **7**



Brazil case study



(a) risk-averse company



(b) risk-seeking company

- 40%+ PSFs in the southern part
- 6 ports in the north do not attract much soybean exports
- Improve transportations infrastructure in north

Brazil case study

Averaged performance measures for the Brazil case

Strategy	No. of PSF	ROI	PSF usage	Profit (million $)		PHL(million $)		Transportation cost (million $)		Subsidy (million $)
				Comp.	Farmer	Comp.	Farmer	Comp.	Farmer	
Risk A.	11	4.6	1.00	501	6611	68	343	468	1319	0
Risk S.	15	3.7	0.95	549	6757	86	297	570	1168	30.7
Comparison (%)	36	-20	-5	9.6	2.2	-6.8		-2.8		N/A

- Risk seeking company earns **9.6% more profit** than the risk-averse company by **investing 36% more** in building PSFs.
- **6.8%** lower PHL, **2.8%** lower transportation cost under 15 PSFs
- ROI (return of investment) is dropped by **20%**

Related Publications

- An, K. and Ouyang, Y. (2016) “Robust grain supply chain design considering post-harvest loss and harvest timing equilibrium.” Transportation Research Part E, 88: 110–128.
- Bai, Y., Hwang, T., Kang, S. and Ouyang, Y. (2011) “Biofuel refinery location and supply chain planning under traffic congestion.” Transportation Research Part B, 45(1): 162–175.
- Nourbakhsh, S.M., Bai, Y. Maia, G., Ouyang, Y. and Rodriguez, L.F. (2016) “Grain supply chain network design and logistics planning for reducing post-harvest loss.” Biosystems Engineering, 151: 105–115.
- Hajibabai, L and Y. Ouyang. (2013) “Integrated planning of supply chain networks and multimodal transportation infrastructure expansion: Model development and application to the biofuel industry.” Computer-aided Civil and Infrastructure Engineering, 28(4): 247-259.

Agricultural land use competition (biofuel vs. food) and government regulations

Biofuel Industry

- Bio-ethanol
  - E10: 10% ethanol + 90% gasoline
  - E85: 85% ethanol + 15% gasoline

- Feedstock
  - Sugarcane
  - Corn
  - Cellulosic biomass (corn stover, miscanthus, switchgrass, wood, etc.)

- "Farm"-to-Pump Life Cycle

Harvest

Transport feedstocks

Production/Blend

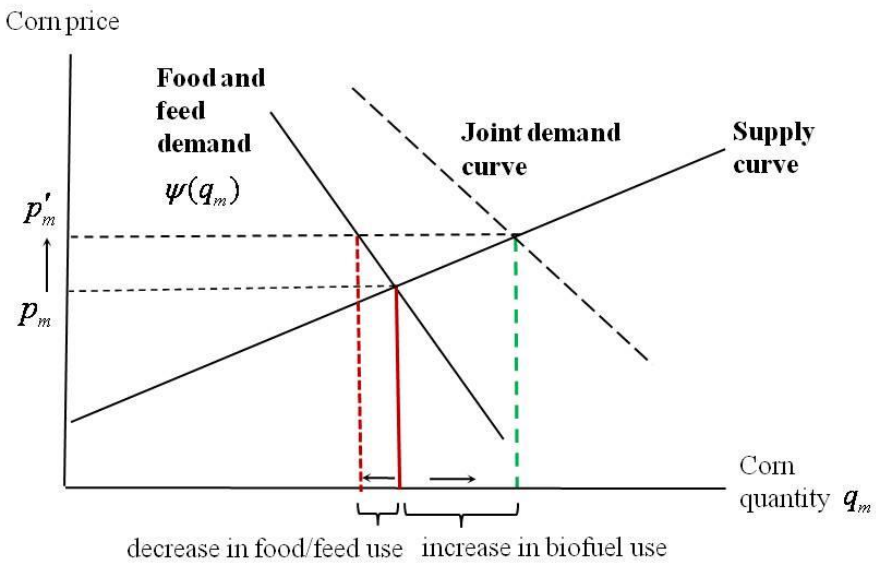
Transport biofuel

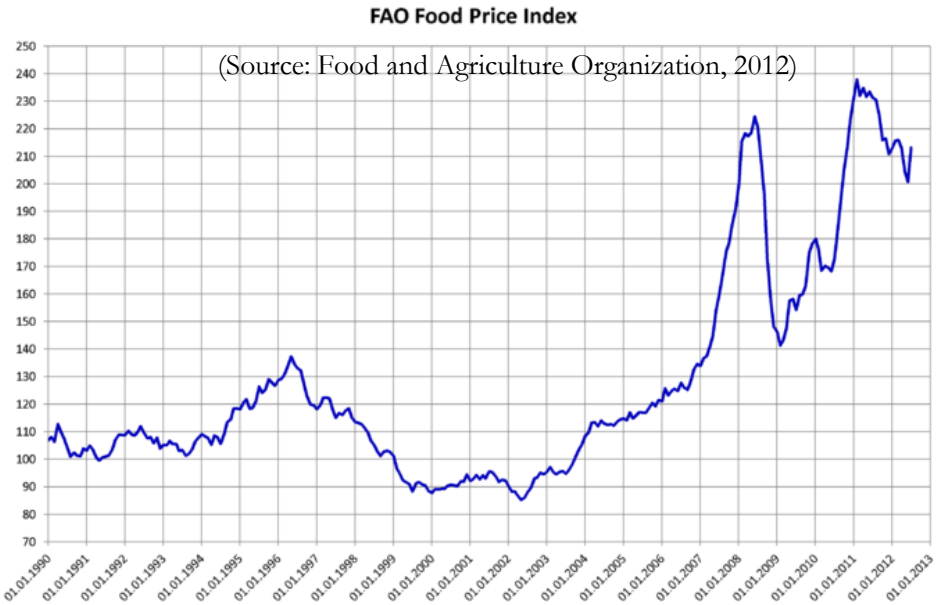
Consumption

Key Issues of the Biofuel Industry

Food vs. Fuel Debate

- Since 2010, fuel has become the No. 1 use of corn in the U.S. *(Cappiello & Apuzzo, 2013)*
- Significant impacts on food price and social welfare *(Walsh et al., 2003; Rajagopal et al., 2009; Johansson & Azar, 2007; Chen et al., 2010)*





Key Issues of the Biofuel Industry

Marginal Land

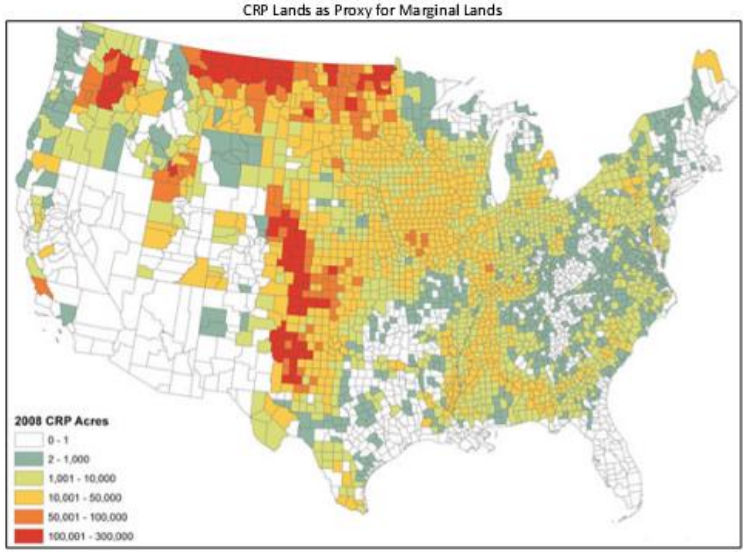
- Low yield for food production (due to access, water, topography, and environmental restrictions)
- Can be reclaimed at a cost

Abandoned Mine in Montana

Source: USGS

- Suitable for large-scale cellulosic biomass production
  - U.S. has 22 million hectares marginal land
  - Potential yield of about 377 million dry tons of biomass per year *(Perlack et al., 2005)*

CRP Lands as Proxy for Marginal Lands

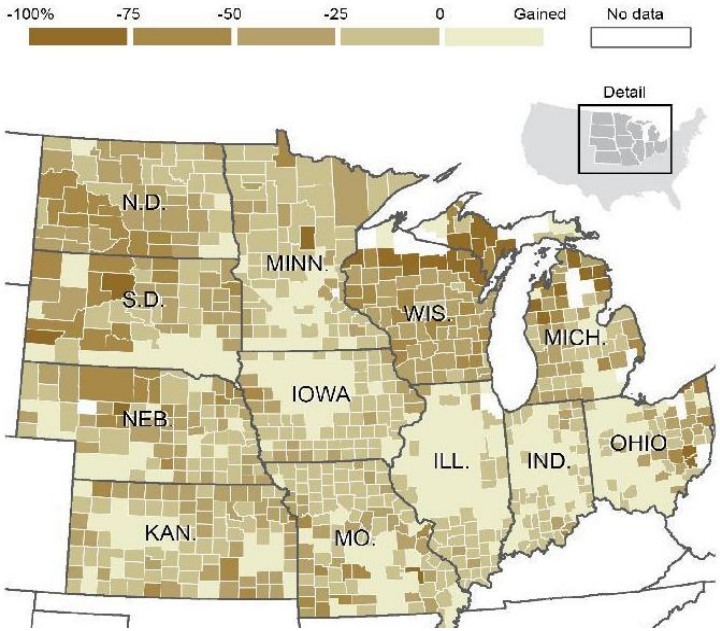


Key Issues of the Biofuel Industry

<u>Environmental Sustainability</u>

- Marginal (or virgin) land serves as a source of environmental conservation (e.g., CO2 sequestration, habitat preservation, soil productivity restoration)
- In the early 2000s, the government pays about $70 per acre for conserved land through the Conservation Reserve Program (CRP)

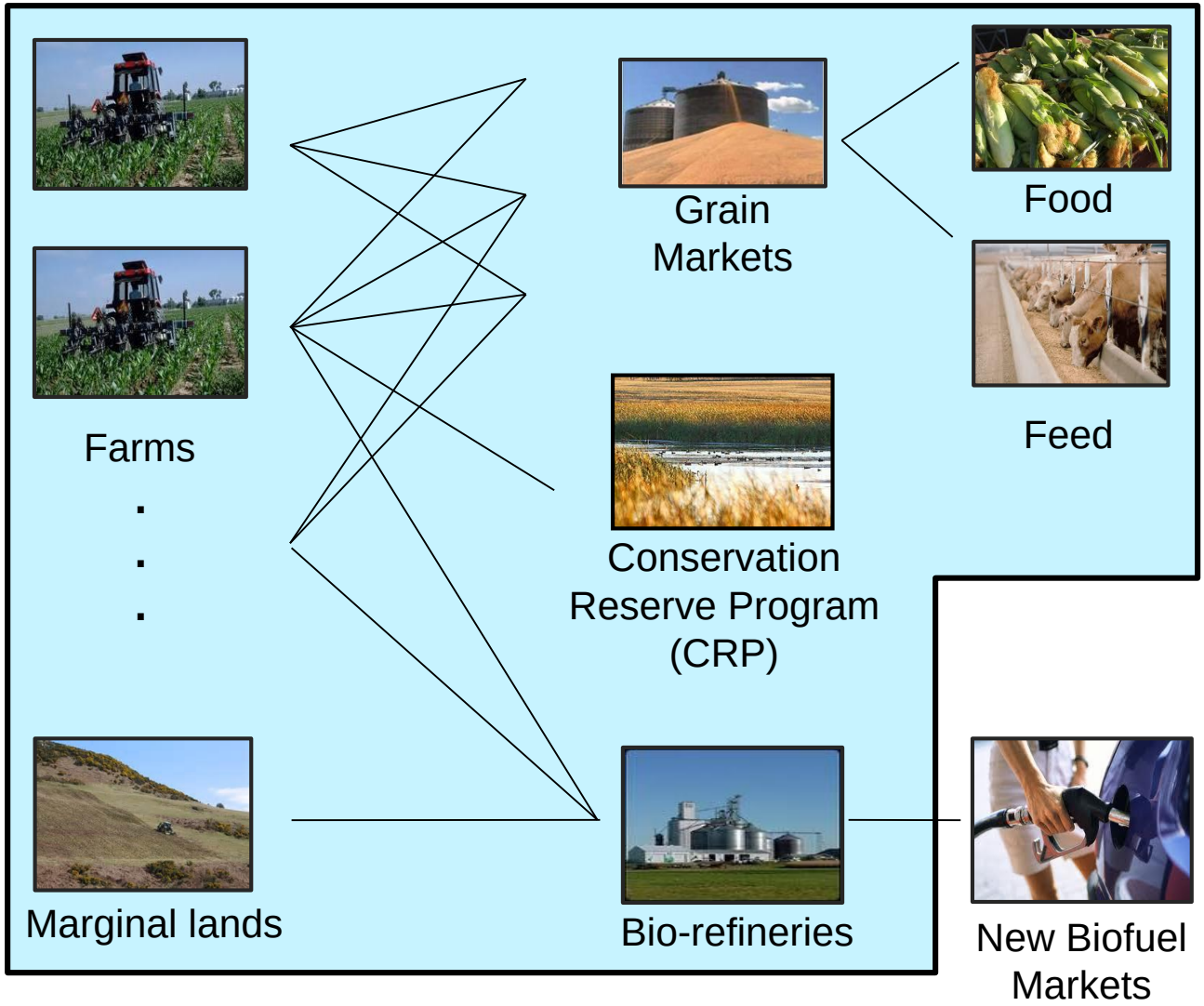
- Since 2007, 2 million hectares of conserved land has been reclaimed
- Utilization of conserved land can cause environmental hazards (e.g., soil erosion and pollution from fertilizer runoff)



Percentage loss of conserved land 2006-2012

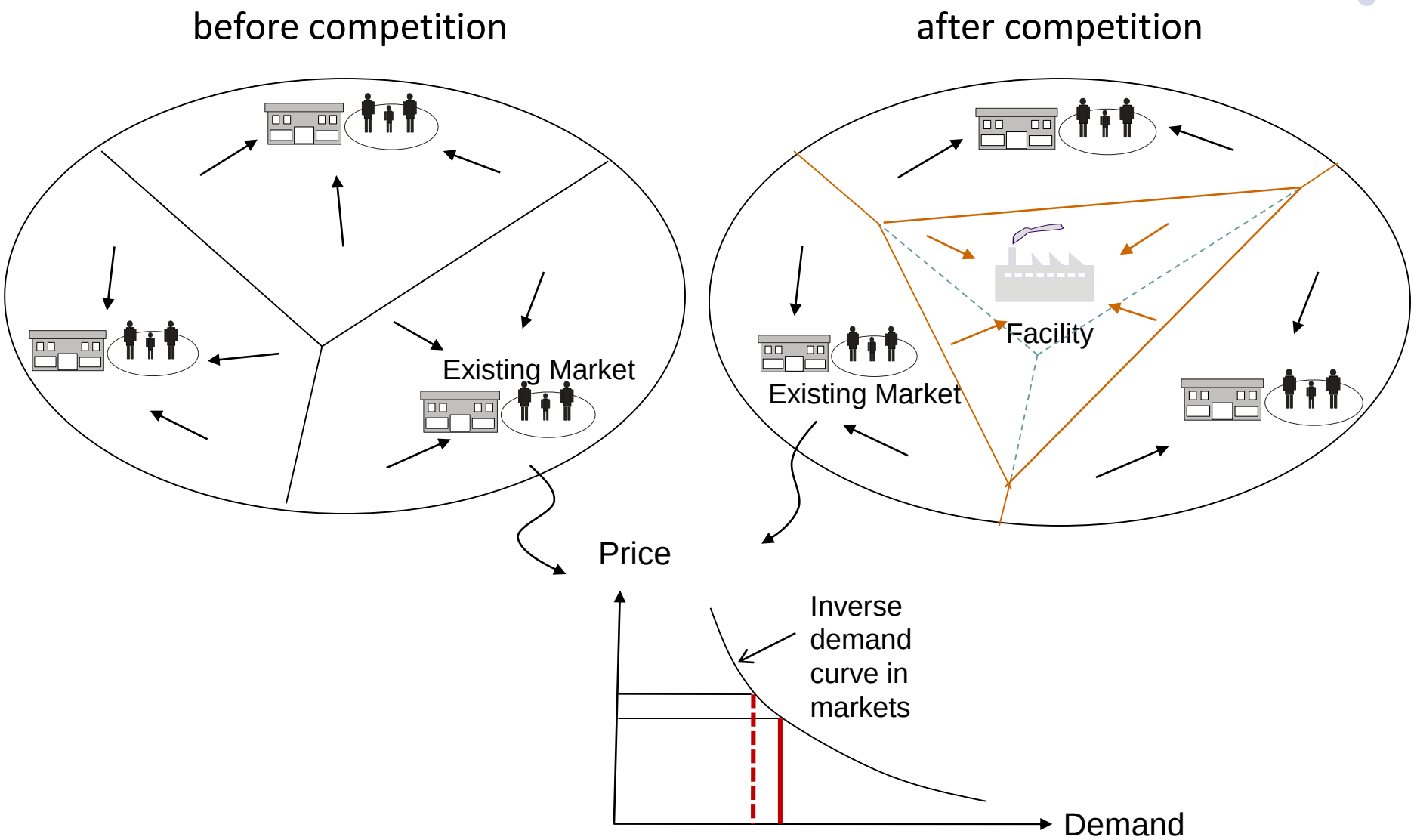
Soil erosion in a cornfield recently converted from pasture near Lineville, Iowa (AP Photo/Charlie Riedel)

Competitive Biofuel Supply Chain



- Biofuel manufacturer:
  - Refinery location
  - Feedstock procurement price
- Farmer
  - Land investment (marginal land reclamation)
  - Land allocation and crop sales
- Government
  - Land use regulation
  - CRP program
- Markets
  - Fuel, food, farmland price at equilibrium

Spatial Equilibrium



Key Questions

- How to quantify the impacts of biofuel industry development on agricultural **land use patterns** (i.e., for energy production, food production and environmental conservation) and social welfare?

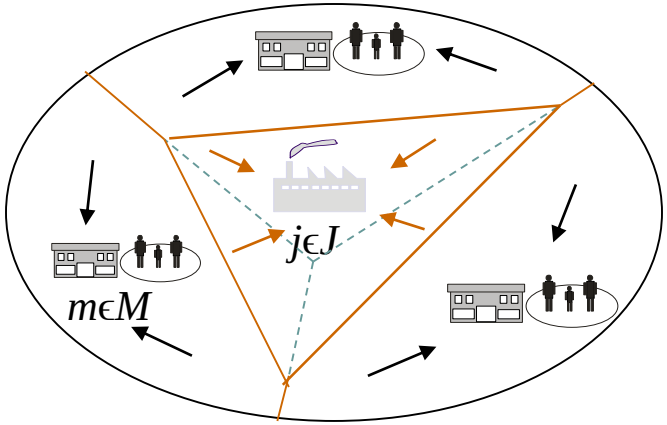
- How to strategically plan the **biofuel supply chain in a competitive environment** where stakeholders (biofuel manufacturer, farmers, etc.) make independent decisions?

- How should **government design policies** to stimulate the growth of the biofuel industry while, at the same time, protect food security and environmental sustainability?

Notation

- Parameters

I, J, M : sets of farms, candidate refinery locations, and existing local food markets
d_{ij} : transportation cost from farm i to a refinery at j
d_{im} : transportation cost from farm i to market m
g_i: existing land owned by farmer $i \in I$
α : total land use percentage for growing energy biomass
$S(\cdot)$: supply curve for marginal land
c_j : maximum capacity of a refinery at location $j \in J$
h_j : fixed cost for building a refinery at location $j \in J$
$p_r(\cdot)$: elastic price for CRP land
$p_e(\cdot)$: biofuel market price
$\psi_m(\cdot)$: inverse demand function for food at market $m \in M$



- Decision variables

x_j : binary variable for refinery construction at location j p_j : biomass procurement price at location j	Industry decision
f_{ij} : land for growing biomass for sales from farm i to a refinery at j f_{im} : land for growing food crops for sales from farm i to a food market at m r_i : land enrolled in the CRP for farmer i	Farmer decision
p_l : market price for marginal land s_l : total utilization of reclaimed marginal land	Market equilibrium

Stackelberg Leader-Follower Game

- **Farmer** *i* maximizes its own sales profit, given refinery location and biomass price

revenue from food crops

revenue from energy biomass

$$\underset{\widehat{\mathbf{f}}_i,\,\widehat{r}_i \geq 0}{\text{maximize}} \quad \sum_{m\in M}\left[\psi_m\left(\widehat{f}_{im}+\sum_{i'\in I\setminus\{i\}} f_{i'm}\right)-d_{im}\right]\widehat{f}_{im}+\sum_{j\in J}\left(p_j-d_{ij}\right)\widehat{f}_{ij}$$

$$+\,p_r\left(\widehat{r}_i+\sum_{i'\in I\setminus\{i\}} r_{i'}\right)\widehat{r}_i$$

revenue from CRP

$$\text{subject to} \quad \widehat{f}_{ij}+\sum_{i'\in I\setminus\{i\}} f_{i'j} \leq c_j\,x_j, \quad \forall\, j \in J$$

refinery capacity constraints

$$\sum_{k\in M\cup J}\widehat{f}_{ik}+\widehat{r}_i+\sum_{i'\in I\setminus\{i\}}\left(r_{i'}+\sum_{k\in M\cup J} f_{i'k}\right) \leq s_\ell+\sum_{i'\in I} g_{i'}$$

land acquisition and energy use limit

$$\text{and} \quad \sum_{j\in J}\left(\widehat{f}_{ij}+\sum_{i'\in I\setminus\{i\}} f_{i'j}\right) \leq \alpha\left[\widehat{r}_i+\sum_{i'\in I\setminus\{i\}} r_{i'}+\sum_{k\in M\cup J}\left(\widehat{f}_{ik}+\sum_{i'\in I\setminus\{i\}} f_{i'k}\right)\right].$$

Stackelberg Leader-Follower Game

- **Biofuel company** aims at maximizing its profit

$$\max_{\boldsymbol{x},\,\boldsymbol{p}_J \geq 0} \quad \overbrace{p_e(\mathbf{f}_J)\sum_{j\in J}\sum_{i\in I} f_{ij}}^{\text{revenue from ethanol sales}} - \overbrace{\sum_{j\in J} h_j x_j}^{\text{fixed cost}} - \overbrace{\sum_{j\in J}\left(p_j \sum_{i\in I} f_{ij}\right)}^{\text{feedstock procurement}}$$

$$s.t. \quad \sum_{i\in I} f_{ij} \leq c_j x_j, \forall j \in J$$

$$x_j \in \{0,1\}, \forall j \in J$$

- Marginal **land market** clearing condition

$$0 \leq p_\ell \perp S^{-1}(p_\ell) + \sum_{i\in I} g_i - \sum_{i\in I}\left(\sum_{k\in M\cup J} f_{ik} + r_i\right) \geq 0,$$

Government Land Use Policy

- Hard for the **government** to impose land use constraints directly on independent farmers
- We propose a cap-and-trade mechanism (Robert, 2001; Zhao, et al., 2010; Chen, et al., 2011) to provide incentives on farmland use
  - Farmers receive an initial allowance (i.e., a percentage of its owned + acquired land) for growing biomass

$$\alpha: \quad \text{land use allowance factor}$$

  - Government restricts the total land use for biofuel by a cap

$$E = \alpha * (\text{total farm land})$$

  - Farmers can trade allowance, i.e., those who need more allowance to grow extra biomass have to purchase from others with surplus

$$p_a': \quad \text{land use allowance price}$$

Integrated Model

$$\max_{\mathbf{x},\, \mathbf{p}_J \geq 0} \quad p_e(\mathbf{f}_J) \sum_{j \in} \qquad - \sum_{j \in J} \left(p_j \sum_{i \in I} f_{ij} \right)$$

$$s.t. \quad \sum_{i \in I}$$

$$x_j \in \{0,1\}, \forall j \in J$$

Biofuel company

$$\sum_{j \in J} \left(\right.$$

$$\leq \alpha \left[\widehat{r}_i + \sum \quad r_{i'} + \sum \left(\widehat{f}_{ik} + \sum_{i' \in I \setminus \{i\}} f_{i'k} \right) \right]$$

Government

$$\underset{\widehat{\mathbf{f}}_i, \widehat{r}_i \geq 0}{\text{maximize}} \quad \sum_{m \in M} \left[\psi_m \left(\widehat{f}_{im} + \sum \quad f_{i'm} \right) - d_{im} \right] \widehat{f}_{im} + \sum_{j \in J} (p_j - d_{ij}) \widehat{f}_{ij}$$

$$+ p_r \left(\right.$$

$$\text{subject to} \quad \widehat{f}_{ij} + \quad i' \in$$

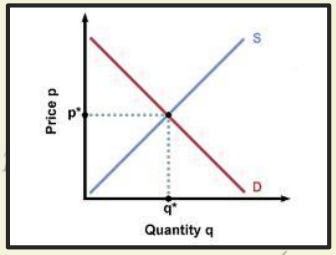
$$\sum_{k \in M \cup J} f_{ik} + r_i + \sum_{i' \in I \setminus \{i\}} \left(r_{i'} + \sum_{k \in M \cup J} f_{i'k} \right) \leq s_\ell + \sum_{i' \in I} g_{i'}$$

$$\text{and} \quad \sum_{j \in J} \left(\widehat{f}_{ij} + \sum_{i' \in I \setminus \{i\}} f_{i'j} \right) \leq \alpha \left[\widehat{r}_i + \sum_{i' \in I \setminus \{i\}} r_{i'} + \sum_{k \in M \cup J} \left(\widehat{f}_{ik} + \sum_{i' \in I \setminus \{i\}} f_{i'k} \right) \right].$$

Farmers

$$0 \leq$$

$$S^{-1}(p_\ell) + \sum \quad \sum \left(\sum_{k \in M \cup J} f_{ik} + r_i \right) \geq 0,$$



Market equilibrium

Existence of Solutions and Equivalence

Theorem 1: The base model and the cap-and-trade have equivalent solutions, yielding the same land allocations for the farmers, if the total cap is properly selected; i.e.,

$$E \triangleq \sum_{i \in I} \sum_{j \in J} f_{ij}^*$$

Theorem 2: Optimal equilibrium solution exists among the farmers for any refinery locations and prices, if price functions $\psi_m(\cdot), p_r(\cdot), S(\cdot)$ are continuous and satisfy the following:

(a) for every scalar $a \geq 0$, the function $\tau \mapsto \tau\psi_m(\tau + a)$ is concave and $\psi_m'(\tau) \leq 0$;

(b) for every scalar $a \geq 0$, the function $\tau \mapsto \tau p_r(\tau + a)$ is concave and $p_r'(\tau) \leq 0$;

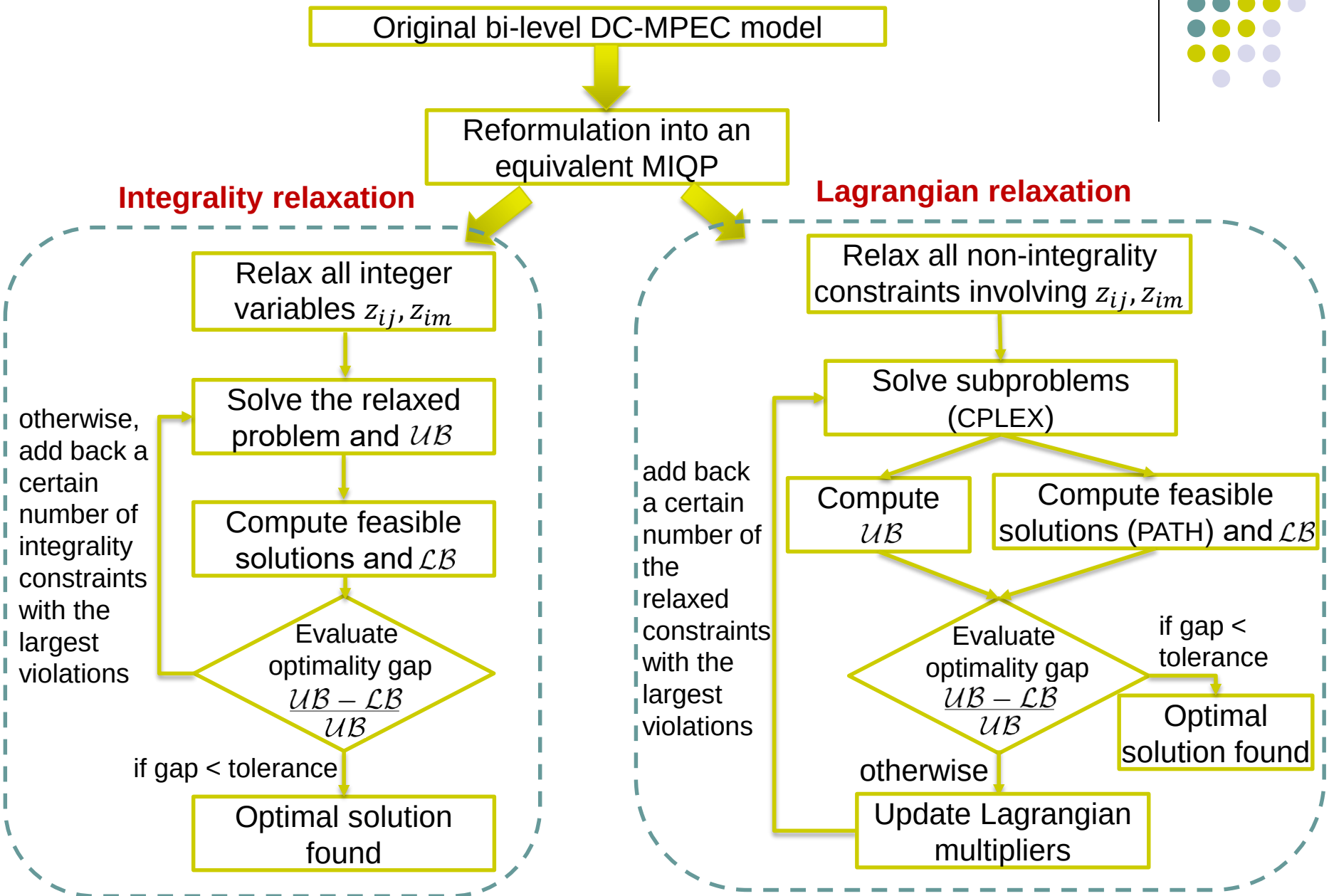
(c) $\limsup_{\tau_r \to \infty} \tau_r \, p_r(\tau_r) + \limsup_{\{\tau_m\}_{m \in M} \to \infty} \sum_{m \in M} \tau_m \, \psi_m(\tau_m) \leq 0$;

(d) either $\limsup_{\tau_r \to \infty} \tau_r \, p_r(\tau_r) + \limsup_{\{\tau_m\}_{m \in M} \to \infty} \sum_{m \in M} \tau_m \, \psi_m(\tau_m) < 0$

or $\max \left\{ \max_{m \in M} \left[\sup_{\tau_m > 0} \psi_m'(\tau_m) \right], \sup_{\tau_r > 0} p_r'(\tau_r) \right\} < 0$,

(e) $S(s)$ is a strictly increasing function for $s \geq 0$ and $S(0) = 0$;

Solution Framework



Solving the DC-MPEC

- Reformulate into a single level MIQP *(Bai et al., 2012)*
  - Convert the lower level problems by KKT conditions
  - Reformulate the complementarity constraints by disjunctive constraints with binary variables
  - Reformulate the bilevel term in the leader's objective into a convex quadratic function of decision variables

$$
\begin{aligned}
\underset{\mathbf{x},\mathbf{p}_J,\mathbf{f},\mathbf{r},p_\ell,p'_a}{\text{maximize}} \quad & p_e\left(\sum_{j\in J}\sum_{i\in I} f_{ij}\right)\sum_{j\in J}\sum_{i\in I} f_{ij} - \sum_{j\in J} h_j x_j - \sum_{i\in I}\left[\sum_{m\in M} f_{im} d_{im} + \sum_{j\in J} f_{ij} d_{ij}\right] - \sum_{j\in J} c_j \rho_j \\
& - p_\ell\left(S^{-1}(p_\ell) + \sum_{i\in I} g_i\right) - \sum_{m\in M}\sum_{i\in I}\left[a_{2m}\left(f_{im} + \sum_{i'\in I} f_{i'm}\right) - a_{1m}\right] f_{im} \\
& + \sum_{i\in I}\left(b_1 r_i - b_2 r_i^2 - b_2 r_i \sum_{i'\in I} r_{i'}\right) \\
\text{subject to} \quad & x_j,\, y_j \in \{0,1\}, \quad \forall j \in J \\
& u_i \in \{0,1\}, \quad \forall i \in I \\
& s,\, v \in \{0,1\}, \\
& z_{ij},\, z_{im} \in \{0,1\}, \quad \forall i \in I,\, j \in J,\, m \in M \\
\text{and} \quad & \text{for all } i \in I, \text{ all } m \in M, \text{ and all } j \in J: \\
& \underline{p}\, x_j \le p_j \le \bar{p}_j x_j \\
& 0 \le f_{im} \le \bar{f}_{im}\, z_{im} \\
& 0 \le d_{im} - a_{1m} + a_{2m}\left(f_{im} + \sum_{i'\in I} f_{i'm}\right) - \alpha p'_a + p_\ell \le \bar{\gamma}_{im}(1 - z_{im}) \\
& 0 \le f_{ij} \le \bar{f}_{ij}\, z_{ij} \\
& 0 \le d_{ij} - p_j + \rho_j + (1-\alpha) p'_a + p_\ell \le \bar{\gamma}_{ij}(1 - z_{ij}) \\
& 0 \le r_i \le \bar{r}_i u_i \\
& 0 \le p_\ell - \alpha p'_a - b_1 + b_2\left(\sum_{i'\in I} r_{i'} + r_i\right) \le \bar{\gamma}^r_i (1 - u_i) \\
& 0 \le \rho_j \le \bar{p}_j y_j \\
& 0 \le c_j x_j - \sum_{i\in I} f_{ij} \le c_j (1 - y_j) \\
& 0 \le p_\ell \le \bar{p}_\ell s \\
& 0 \le S^{-1}(p_\ell) + \sum_{i\in I} g_i - \sum_{i\in I}\left(\sum_{k\in M\cup J} f_{ik} + r_i\right) \le \left(S^{-1}(\bar{p}_\ell) + \sum_{i\in I} g_i\right)(1 - s) \\
& 0 \le p'_a \le \bar{p}'_a v \\
& 0 \le \alpha \sum_{i\in I}\left(\sum_{k\in M\cup J} f_{ik} + r_i\right) - \sum_{i\in I}\sum_{j\in J} f_{ij} \le \alpha\left(S^{-1}(\bar{p}_\ell) + \sum_{i\in I} g_i\right)(1 - v) \\
& \text{and} \quad \text{for all } j \in J, \quad \begin{cases} x_j + y_j \ge 1 \\ x_j \ge z_{ij}, \quad \forall i \in I. \end{cases}
\end{aligned}
$$

Solving the Large Scale MIQP

- The equivalent MIQP
  - Still challenging to solve due to the large number of binary variables from complementarity constraints
  - Solvers such as CPLEX have limited applicability to small problem sizes
  - Customized iterative "relax-and-tighten" algorithms for moderate or large scale problems (e.g., over 2000 complementarity constraints)
    - Integrality relaxation
    - Lagrangian relaxation

Step 1. Let $n = 1$. Initialize sets F_1, R_1, bounds $\mathcal{UB}$, $\mathcal{LB}$.

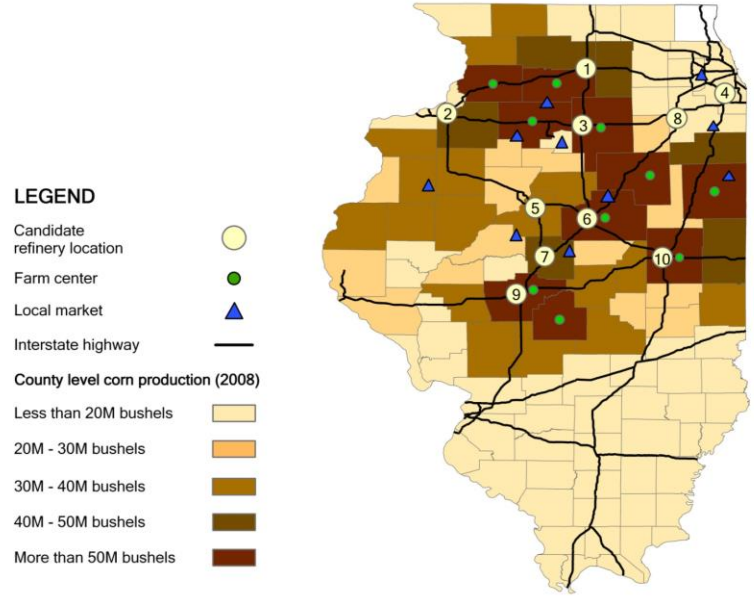
Step 2. In iteration n, solve a relaxed problem with all constraints in set F_n. Update $\mathcal{UB}$ if the current solution provides the tightest upperbound.

Step 3. Compute feasible solutions (lowerbounds) based on the solution $(\mathbf{x}, \mathbf{p_J})$ to the relaxed problem. Update $\mathcal{LB}$ if the current solution provides the best objective value.

Step 4. Compute the optimality gap, i.e., $(\mathcal{UB} - \mathcal{LB})/\mathcal{UB}$. If the gap is below a user defined tolerance, terminate.

Step 5. Inspect the solution of the relaxed problem in step 2 by comparing the violations of the current relaxed constraints in R_n. Choose a certain number (e.g., 10) of constraints $s \subset R_n$ with the highest violations and add them back to the constraint set of the next relaxed problem, i.e., $F_{n+1} = F_n \cup s$ and $R_{n+1} = R_n \backslash s$. Go to step 2.

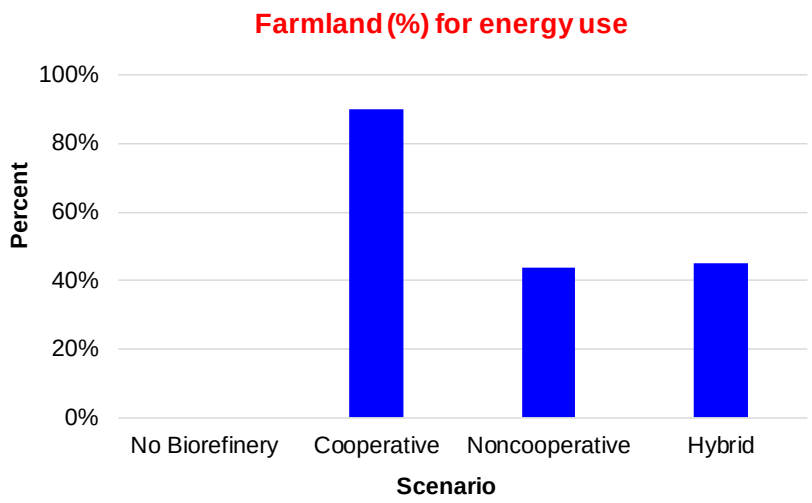
Illinois Case Study

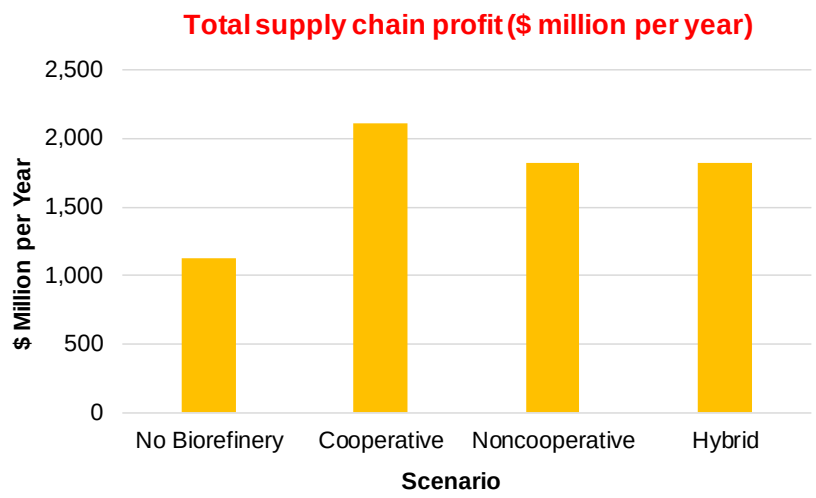


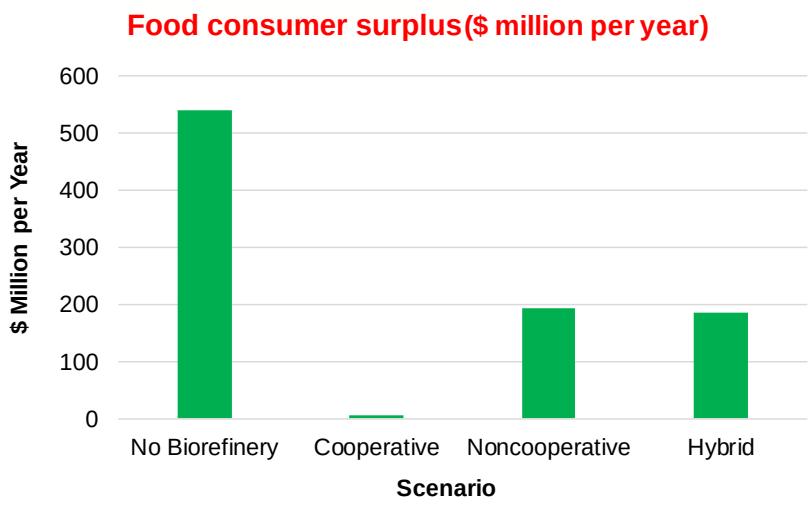
- Illinois
  - Leading state for bio-ethanol production
  - County-level corn production data in 2008 *(Kang et al., 2010)*
  - Projected 17% national ethanol production in 2022 *(ICGA, 2010)*
- Transportation network
  - Candidate refinery locations: major interstate highway intersections *(Bai et al., 2012)*
  - Farms: top corn production counties *(Kang et al., 2010)*
- Key cost coefficients
  - Transport cost = $0.0035/bushel-mile by semi-trailers *(Mcvey et al., 2007; NTAD, 2008)*
  - Prorated refinery cost = $18 million/yr for capacity = 100 million gal/yr *(Bai et al., 2012)*
  - Ethanol production cost = $1.08/gallon *(Brown et al., 2007)*
- Land use parameters $\alpha = 30\%$ $S(s_\ell) = 1.0^{-8} s_\ell$ ($)

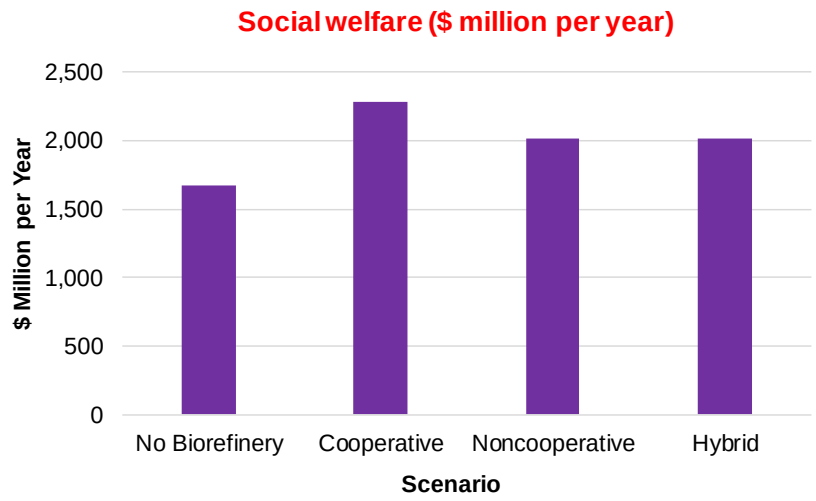
  $p_r\left(\sum_{i\in I} r_i\right) = 4.0 - 2.0^{-8}\sum_{i\in I} r_i$ ($/unit land)
- Two scenarios
  - <u>Benchmark model</u>: no land use regulation (denote as P1)
  - <u>Proposed model</u>: land use regulation (denote as P2)

Impacts of Competition/Cooperations





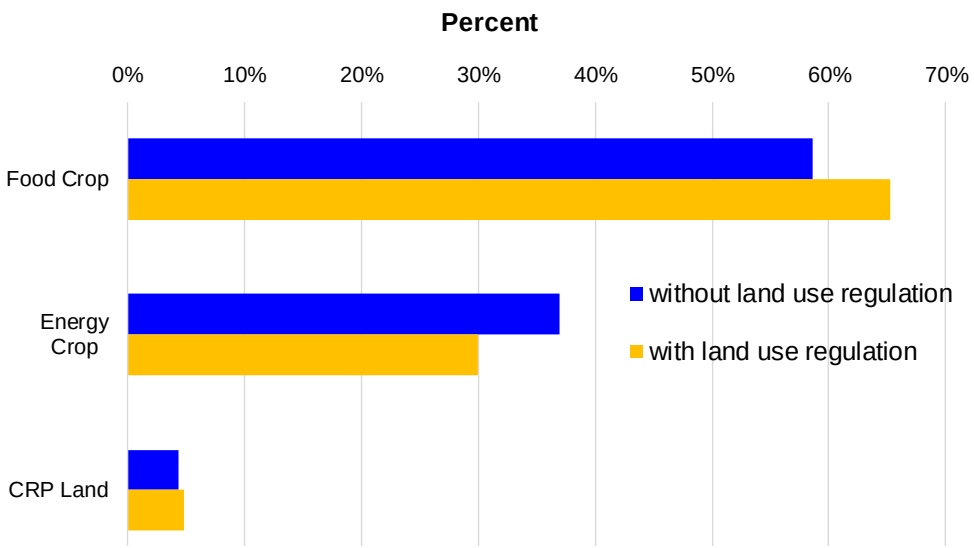




Impacts of Land Use Regulation

(|I|, |J|, |M|)	refinery capacity	scenario	refinery number	farmland use (%)				cost (million $)				revenue (million $)			biofuel industry profit (million $)	social welfare (million $)	pl ($/unit land)	pa ($/unit land)	total permit trading (unit land)
				food crop	energy crop	reseravation	marginal land	total marginal land cost	total transportation cost	total fixed cost	total ethanol production cost	corn sales in food market	ethanol sales in energy market	CRP land revenue					
(10, 10, 10)	25 MGY	P1 (w.o. land use cap)	4	61.0%	32.5%	6.5%	35.9%	238	261	18	847	1174	1988	99	367	2546	1.54	-	-
		P2 (w. land use cap)	4	63.3%	30.0%	6.7%	38.5%	299	208	18	815	1172	1927	96	367	2588	1.73	1.27	219080
	50 MGY	P1 (w.o. land use cap)	2	61.0%	32.5%	6.5%	35.9%	238	218	18	847	1174	1988	99	367	2589	1.54	-	-
		P2 (w. land use cap)	2	63.3%	30.0%	6.7%	38.6%	302	205	18	816	1172	1930	96	367	2592	1.74	1.32	322260
	100 MGY	P1 (w.o. land use cap)	1	61.0%	32.5%	6.5%	35.9%	238	187	18	847	1174	1988	99	367	2620	1.54	-	-
		P2 (w. land use cap)	1	63.3%	30.0%	6.7%	38.6%	302	205	18	816	1172	1930	96	367	2592	1.74	1.32	348663
(30, 30, 30)	25 MGY	P1 (w.o. land use cap)	8	58.8%	36.8%	4.4%	21.1%	348	236	36	1681	2804	4108	177	886	5447	2.64	-	-
		P2 (w. land use cap)	7	65.2%	30.0%	4.8%	20.3%	321	255	32	1362	2968	3422	185	813	5262	2.54	0.13	478835
	50 MGY	P1 (w.o. land use cap)	4	58.6%	37.0%	4.3%	21.2%	349	229	36	1693	2797	4133	177	890	5460	2.64	-	-
		P2 (w. land use cap)	4	65.2%	30.0%	4.8%	20.4%	322	251	36	1362	2968	3423	185	809	5261	2.54	0.14	544973
	100 MGY	P1 (w.o. land use cap)	2	58.6%	37.0%	4.3%	21.2%	349	225	36	1693	2797	4133	177	891	5464	2.64	-	-
		P2 (w. land use cap)	2	65.2%	30.0%	4.8%	20.4%	323	249	36	1362	2969	3424	185	809	5264	2.54	0.15	589969

- Impacts of land use regulation
  - Government: Less land used to produce energy crops; the regulation cap is binding
  - Farmers: Willing to pay for allowance, $p_a > 0$, so biomass production is profitable
  - Biofuel manufacturer: Lower profit and lower producer surplus
  - Markets: Higher food consumer surplus



(30 farms,30 refinery locations,30 food markets)

Related Publications

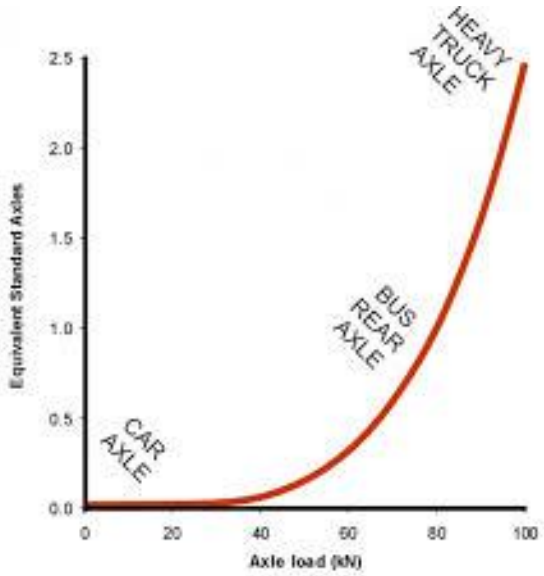
- Bai, Y., Ouyang, Y. and Pang, J.S. (2016) “Enhanced models and improved solution for competitive biofuel supply chain design under land use constraints.” European Journal of Operational Research, 249(1):281–297.
- Wang, X., Lim, M.K. and Ouyang, Y. (2015) “Infrastructure deployment under uncertainties and competition: The biofuel industry case.” Transportation Research Part B, 78: 1-15.
- Wang, X.L., Ouyang, Y., Yang, H. and Bai, Y. (2013) “Optimal biofuel supply chain design under consumption mandates with renewable identification numbers.” Transportation Research Part B, 57: 158-171.
- Bai, Y., Ouyang, Y. and Pang, J.S. (2012) "Biofuel supply chain design under competitive agricultural land use and feedstock market equilibrium." Energy Economics, 34 (5): 1623-1633.

Joint planning of agricultural supply chain and roadway infrastructure rehabilitation

Introduction

- 11% of freight truck traffic causes over 99% of wear-and-tear on U.S. roads
  - Road damage rises with the fourth power of vehicle axle load

    (http://pavementinteractive.org)
  - One axle of 10 tons (heavy truck) is 160,000 times more damaging than an axle of 0.5 tons (passenger car)



- Growing agricultural freight traffic accelerates pavement deterioration
  - Higher deterioration rate and thus higher roadway maintenance needs
  - Poorer travel quality for public road users as well as freight carriers themselves

alert.air-worldwide.com

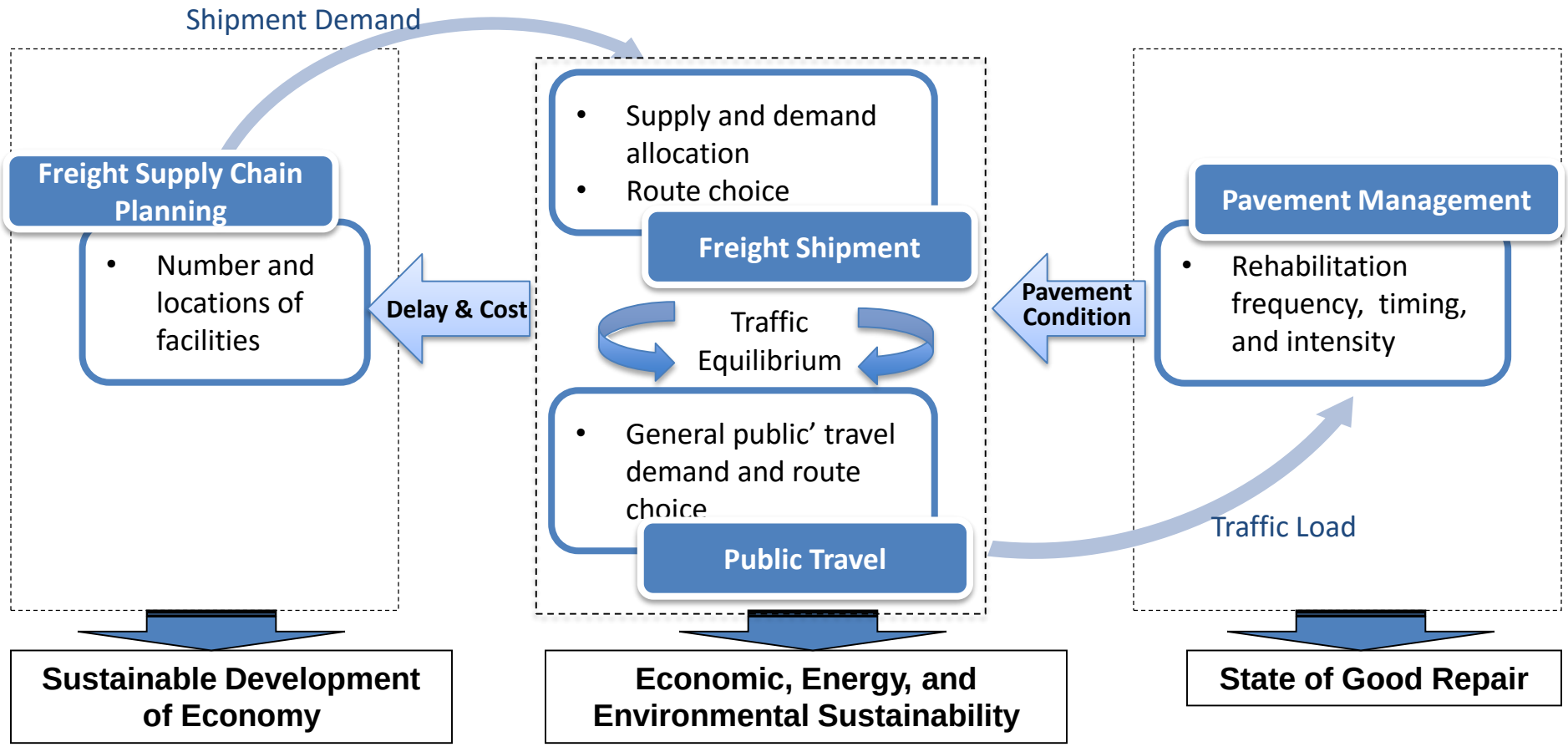
Example

"I wish I was exaggerating when I say that there are roads that are being destroyed and that have been literally turned into mud and made impassable for all motorists …"

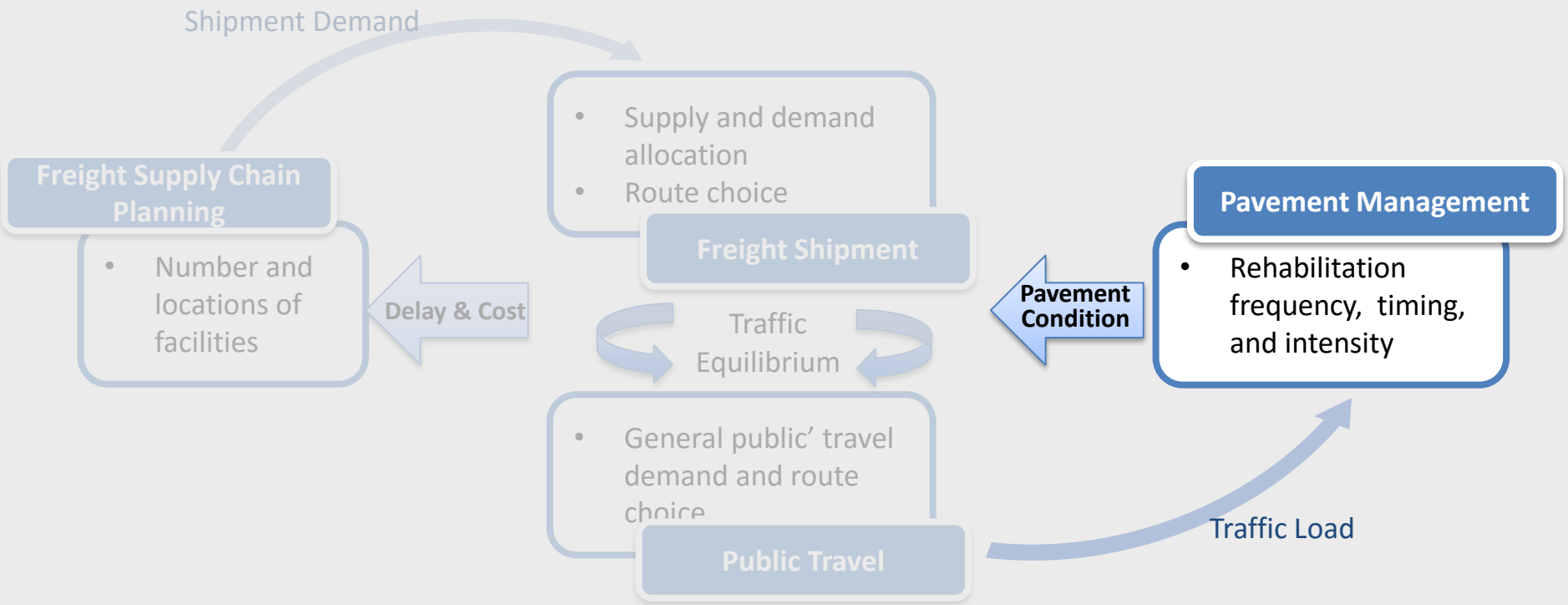
Pennsylvania Department of Environmental Protection's Secretary John Hanger told Pittsburgh's National Public Radio in 2010
http://wduqnews.blogspot.com/2010/09/fracnet-and-trashnet-to-expand.html

Problem Statement

- Planning of new freight facility locations should simultaneously account for the major impacts on traffic operations and network infrastructure
  - Congestion and pollution
  - Pavement deterioration



Problem Statement



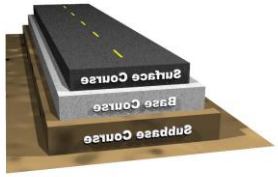
Pavement Rehabilitation

- Pavement deterioration
  - Factors: structure, traffic, environment
  - Distresses: fatigue cracking, thermal cracking, rutting, etc
  - Performance function: roughness measure, e.g., IRI
- Current practice
  - Rehabilitate after functional or structural failure
  - Asphalt concrete (AC) overlay

Traffic load

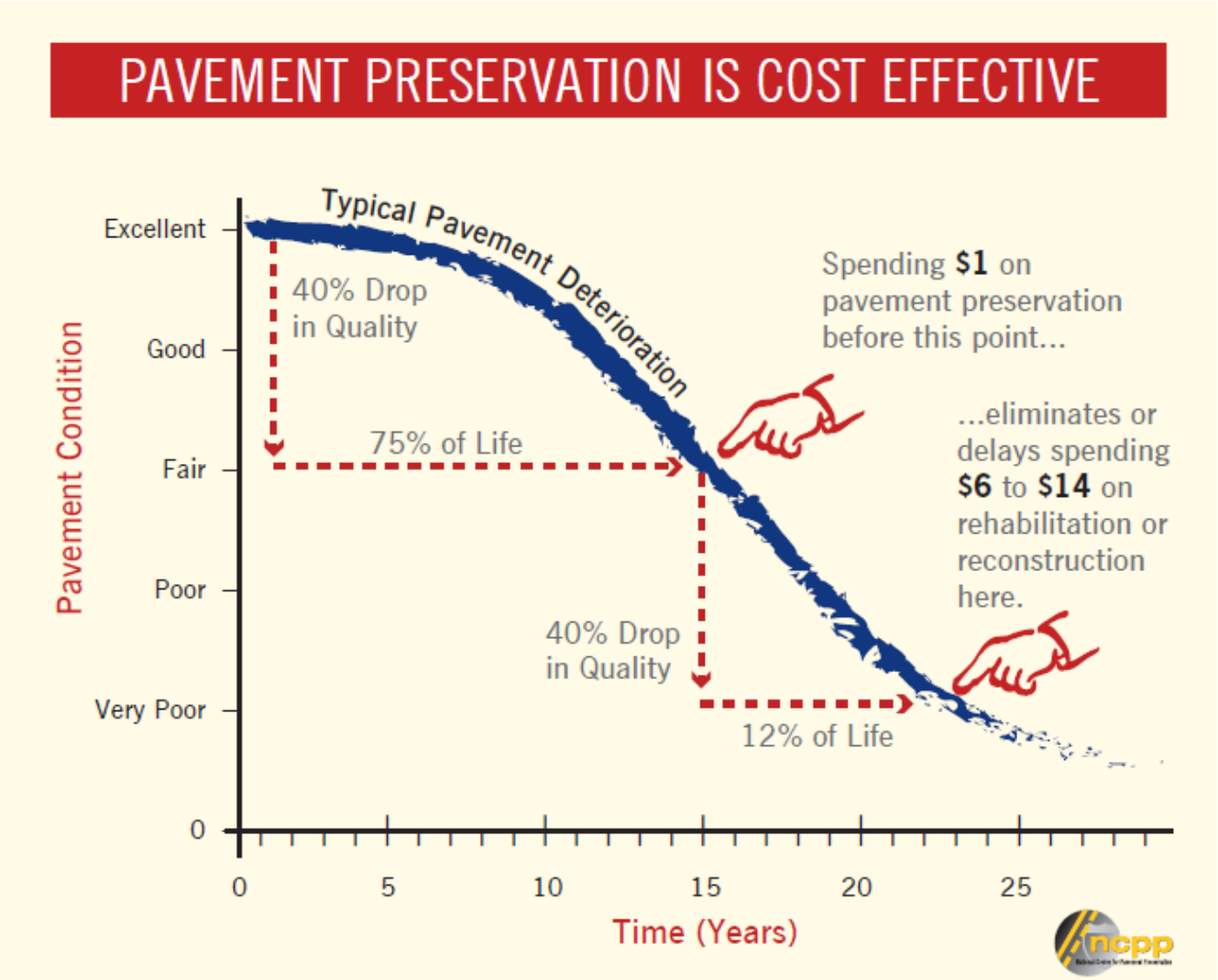
Environment

Design & material



Distress development & performance deterioration over time

(AASHTO 2013; Bai et al., 2014)



Source: National Center for Pavement Preservation.

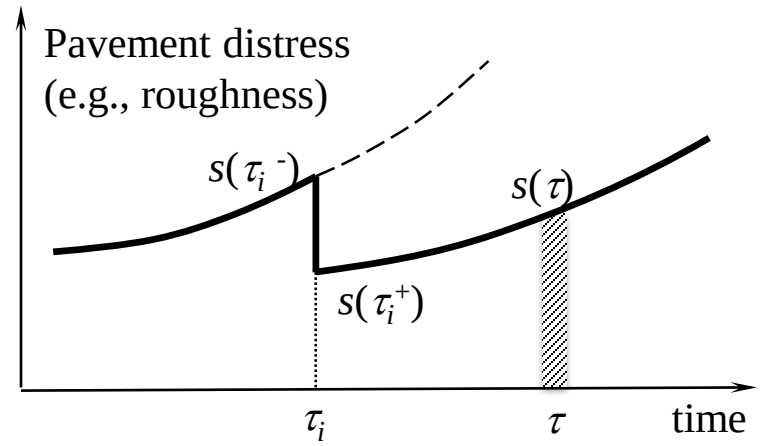
Pavement Model

- For an existing pavement facility of length l, given traffic load

  x = freight truck flow

  z^t = truck flow in background traffic

  z^p = pax car flow in background traffic



- Pavement deteriorates exponentially over time (Ouyang, 2007)
  - Traffic load by type
  - Age

$$s(\tau) = s(\tau_i^+)e^{b(x,z^p,z^t)(\tau-\tau_i^+)}, \forall \tau \in (\tau_i^+, \tau_{i+1}^-)$$

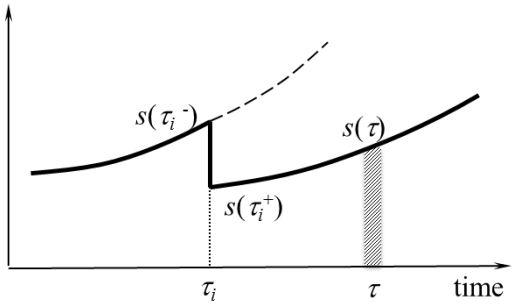
$$b(x, z^p, z^t) = e_0(x + z^t) + e_1 z^p + e_2,\ e_0 \gg e_1$$

- Rehabilitation with intensity w (resurfacing thickness) improves pavement condition

  (Ouyang and Madanat, 2004)

$$\Delta s(s(\tau_i^-), w) := s(\tau_i^-) - s(\tau_i^+) = \min\left\{\frac{g_1 w}{g_2 + g_3/s(\tau_i^-)}, g_1 s(\tau_i^-)\right\}$$

Cost Model



- Both freight and public traffic experiences cost and delay :

  Vehicle operating cost per unit distance is approximately proportional to pavement roughness (Ouyang and Madanat, 2006; Tan et al., 2012):

$$c_r(s(\tau)) = c_1 s(\tau) + c_2, \ \forall \tau$$

  Delay cost depends on factors such as traffic volume and road capacity; we assume travel time per vehicle follow BPR function (1970):

$$t(x, z^p, z^t) = t_0 \left\{ 1 + \alpha \left(\frac{x + z^p + z^t}{\psi} \right)^\beta \right\}$$

  Total cost (per vehicle):

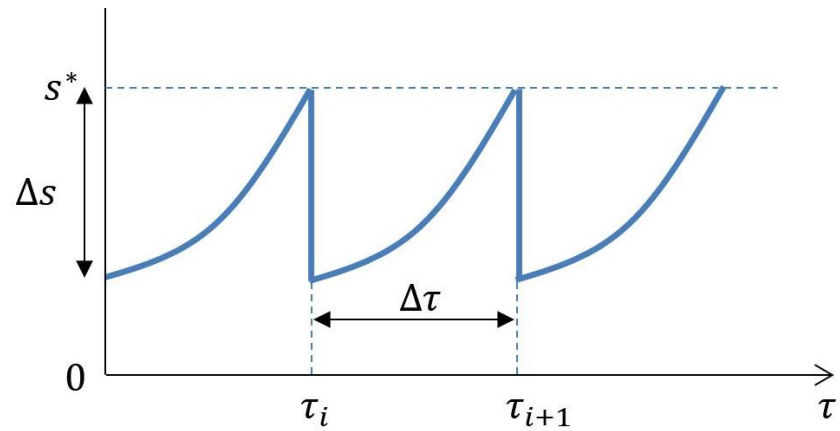
$$C(x, z^p, z^t, s(\tau)) = c_0 t(x, z^p, z^t) + l(c_1 s(\tau) + c_2)$$

- The agency cost for a pavement rehabilitation activity is assumed to be proportional to rehabilitation intensity:

$$M_0(w) = l m_1 w$$

Optimal Rehabilitation

Lemma 1. (Ouyang and Madanat, 2006) Optimal pavement roughness trajectory follows a saw-tooth pattern, whereas rehabilitation is conducted at certain intensity w^* whenever the roughness reaches a trigger value s^*, where



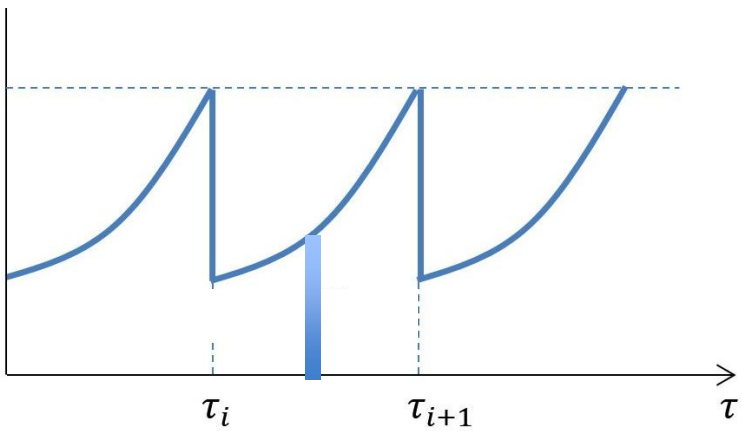
$$w^* = g_2 \frac{r m_1 g_3}{c_1 g_1 \cdot (x + z^p + z^t) + (b(x, z^p, z^t) - r) m_1 g_2} + g_3$$

$$s^* = \frac{r m_1 g_3}{c_1 g_1 \cdot (x + z^p + z^t) + (b(x, z^p, z^t) - r) m_1 g_2}$$

$$\Delta s^* = \frac{g_1 w^*}{g_2 + g_3 / s^*} \qquad \Delta\tau^* = \ln\left(\frac{g_1}{1 - g_1}\right) \Big/ b(x, z^p, z^t)$$

Pavement–Traffic Steady-State

- If the pavement condition perceived by the travelers equals the long-run average condition over time (ignore dynamics), then
  - Actual average vehicle operating cost per unit time equals the traveler's perception
  - Traveler's route choice would form a steady traffic load even when pavement condition contributes to the cost

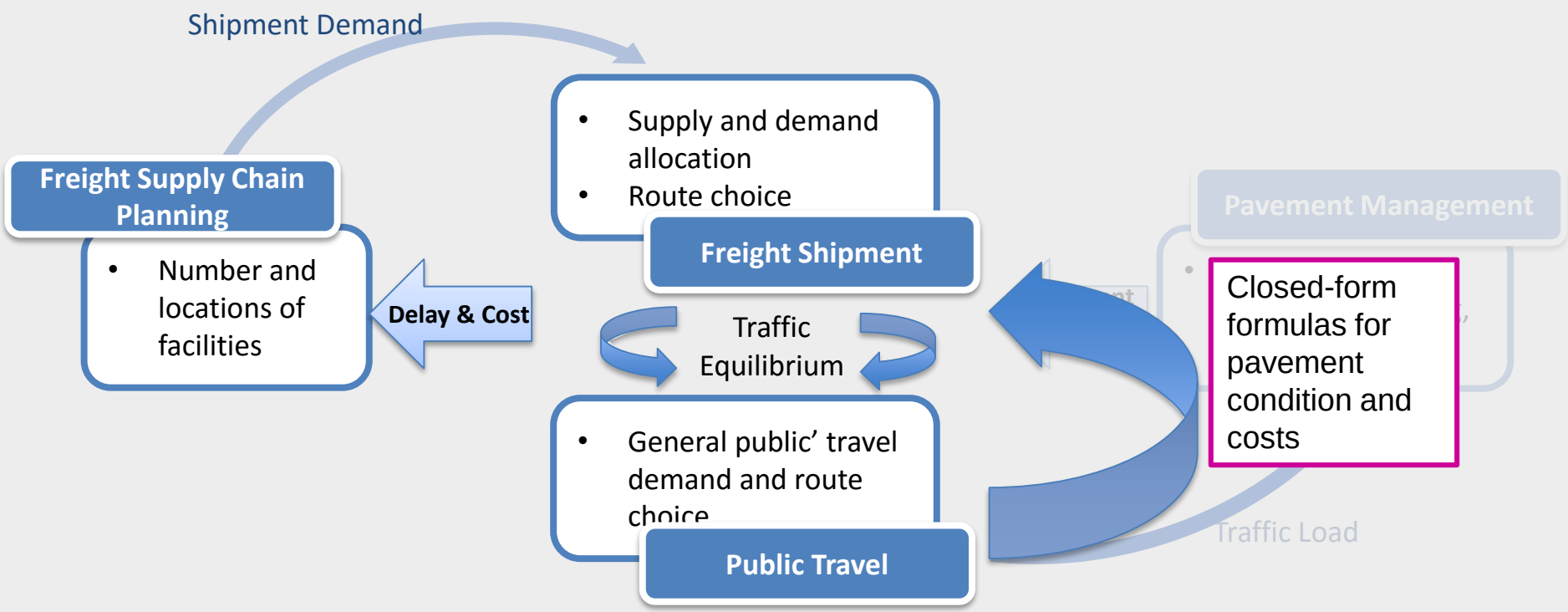


Lemma 2. (Hajibabai et al., 2014) Under the optimal rehabilitation plan, the total traveler and agency costs per unit time can be expressed in closed-form as functions of the traffic load, as follows,

$$C(x, z^p, z^t) = c_0 t(x, z^p, z^t) + l\left(\frac{c_1 r m_1 g_3 (2g_1 - 1)\ln^{-1}(\frac{g_1}{1-g_1})}{c_1 g_1 \cdot (x + z^p + z^t) + (b(x, z^p, z^t) - r) m_1 g_2} + c_2\right)$$

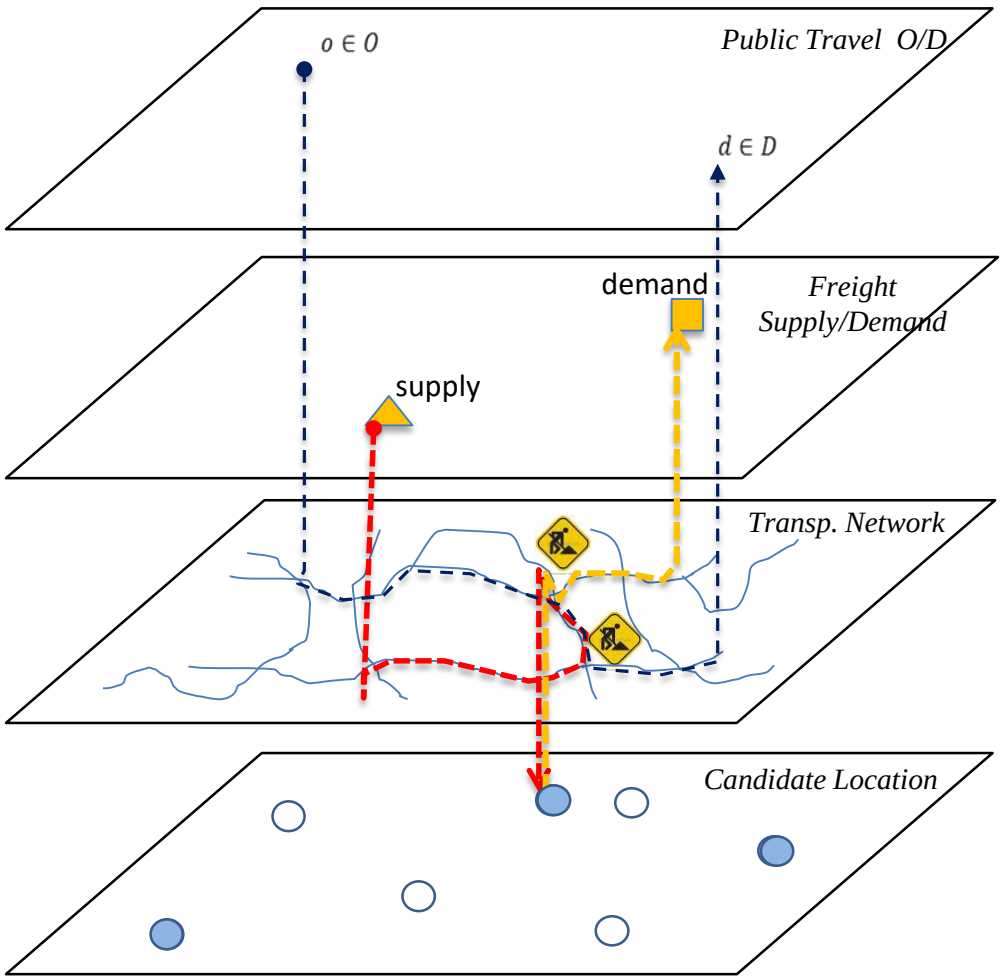
$$M(x, z^p, z^t) = \frac{b(x, z^p, z^t) l g_3 m_1 \ln^{-1}(\frac{g_1}{1-g_1})\left(g_2 r m_1 + c_1 g_1 \cdot (x + z^p + z^t) + (b(x, z^p, z^t) - r) m_1 g_2\right)}{c_1 g_1 \cdot (x + z^p + z^t) + (b(x, z^p, z^t) - r) m_1 g_2}$$

Problem Statement



Model: Joint Optimization

- Given
  - Transportation network (including pavement information)
  - Freight supply/demand location and quantity
  - Public travel O/D demand
  - Candidate freight facility location, capacity, and construction cost
- Decision
  - Freight processing facility location
  - Freight shipment routes
  - Public traffic equilibrium
  - (Pavement rehabilitation plan)



Model: Joint Optimization

- Notations

$G(V,\ A)$: highway network

$I,\ K \subset V$: sets of freight supply and demand locations

Q_i, Q_k: total amount of raw material supply and total amount of final product demand

γ: volume / weight conversion factor from raw material flow to product flow

$J \subset V$: set of candidate locations for production facilities

h_j, η_j: maximum capacity and fixed construction cost of production facility $j \in J$

$Q_u^{p,od}, Q_u^{t,od}$: background traffic demand generated on node $u \in V$ associated with $od \in OD$,

$$\text{where} \begin{cases} Q_u^{p,od}, Q_u^{t,od} > 0, & \text{if } u \text{ is the origin node of } od \\ Q_u^{p,od}, Q_u^{t,od} < 0, & \text{if } u \text{ is the destination node of } od \\ Q_u^{p,od}, Q_u^{t,od} = 0, & \text{O.W.} \end{cases}, \ \forall u \in V,\ od \in OD$$

OD: set of origin - destination pairs of existing traffic in the background

- Key decision variables

y_j: facility location binary variable for $j \in J$

x_a and $z_a^p,\ z_a^t$: freight and background traffic flow on link $a \in A$

b_a: deterioration rate on link $a \in A$

$f_{a,1}, f_{a,2}$: raw material flow and final product flow on arc $a \in A$

$q_{j,1}, q_{j,2}$: net inbound raw material flow and outbound product flow (throughput of facility $j \in J$)

f_a^{od}: background traffic flow associated with od on arc $a \in A$

Model: Joint Optimization

- Upper level: supply chain design and pavement rehabilitation decisions

(a)
$$\underset{\mathbf{y}\in\{0,1\}^{|J|},\,\mathbf{x},\mathbf{z}^p,\mathbf{z}^t,\mathbf{b},\mathbf{f},\mathbf{q}\geq 0}{\text{minimize}} \quad \boxed{\sum_{j\in J}\eta_j y_j + \sum_{a\in A} M_a(x_a,\, z_a^p,\, z_a^t) + \sum_{a\in A}\left(x_a + z_a^p + z_a^t\right) C_a(x_a,\, z_a^p,\, z_a^t)}$$

Facility + pavement + transportation cost

(b)
$$\boxed{b(x, z^p, z^t) = e_0(x + z^t) + e_1 z^p + e_2,}$$

Deterioration rate depends on traffic load

(c)
$$q_{j,1} = \sum_{a\in A_j^+} f_{a,1} - \sum_{a\in A_j^-} f_{a,1},$$

(d)
$$q_{j,2} = \sum_{a\in A_j^-} f_{a,2} - \sum_{a\in A_j^+} f_{a,2},$$

Net inbound raw material flow and outbound product flow (i.e., throughput of facilities)

(e)
$$\boxed{q_{j,1} \leq h_j y_j, \forall j \in J,}$$

Facility location and capacity constraints

(f)
$$\sum_{a\in A_i^+} f_{a,1} - \sum_{a\in A_i^-} f_{a,1} = \begin{cases} Q_i, \forall i \in I, \\ 0, \forall i \in V\backslash(I\cup J), \end{cases}$$

(g)
$$\sum_{a\in A_k^+} f_{a,2} - \sum_{a\in A_k^-} f_{a,2} = \begin{cases} Q_k, \forall k \in K, \\ 0, \forall k \in V\backslash(K\cup J), \end{cases}$$

Flow conservation constraints

(h)
$$\boxed{\gamma q_{j,1} = q_{j,2}, \forall j \in J,}$$

Raw materials and products flow conservation

(i)
$$\boxed{x_a = f_{a,1} + f_{a,2}, \forall a \in A,}$$

Link-path flow relationship

Model: Joint Optimization

- Lower level: user equilibrium (UE):
  - Assignment of public travel demand
  - Reflect individual route choice behavior (under vehicle operating + delay costs)

$$\mathbf{z}^p,\, \mathbf{z}^t \in \underset{\mathbf{f}^p,\, \mathbf{f}^t \geq 0, \boldsymbol{\lambda}}{\operatorname{argmin}} \sum_{a \in A} \left(z_a^p + z_a^t\right) C_a(x_a,\, z_a^p,\, z_a^t) - \sum_{u \in V} \sum_{od \in OD} \left(Q_u^{p,od} + Q_u^{t,od}\right) \lambda_u^{od} \qquad \text{(j)}$$

$$\text{subject to} \quad z_a^p = \sum_{od \in OD} f_a^{p,od} \qquad \text{(k)}$$

$$z_a^t = \sum_{od \in OD} f_a^{t,od} \qquad \text{(l)}$$

$$\sum_{a \in A_u^+} f_a^{p,od} - \sum_{a \in A_u^-} f_a^{p,od} = Q_u^{p,od}, \forall u \in V, od \in OD \qquad \text{(m)}$$

$$\sum_{a \in A_u^+} f_a^{t,od} - \sum_{a \in A_u^-} f_a^{t,od} = Q_u^{t,od}, \forall u \in V, od \in OD \qquad \text{(n)}$$

$$\lambda_u^{od} - \lambda_v^{od} \leq C_a(x_a, z_a^p, z_a^t),\, \forall a = (u, v) \in A,\, od \in OD, \qquad \text{(o)}$$

$$f_a^{p,od},\, f_a^{t,od} \geq 0,\, \forall a \in A,\, od \in OD. \qquad \text{(p)}$$

Solution Approach – Single-Level Reformulation

- KKT-based approach: reformulate our problem into a single-level NLP

$$\underset{\mathbf{y}\in\{0,1\}^{|J|},\mathbf{x},\mathbf{z}^p,\mathbf{z}^t,\boldsymbol{\lambda},\mathbf{f},\mathbf{q}\geq 0}{\text{minimize}} \quad \sum_{j\in J}\eta_j y_j + \sum_{a\in A} M_a(x_a, z_a^p, z_a^t) + \sum_{a\in A}(x_a + z_a^p + z_a^t)\, C_a(x_a, z_a^p, z_a^t) \qquad \text{(q1)}$$

subject to (b)-(e), (f)-(i), (k)-(p), and

$$\lambda_u^{od} - \lambda_v^{od} \leq C_a,\ \forall a = (u,v) \in A, \qquad \text{(r2)}$$

$$C_a \geq C_a(x_a,\ z_a^p,\ z_a^t),\ \forall a \in A. \qquad \text{(r3)}$$

where $\mathbf{C} = \{C_a\}_{a\in A}$ is an auxiliary variable to represent the link user cost, i.e.,

.

$$C_a = \overline{C_a}(x_a,\ z_a^p,\ z_a^t),\ \forall a \in A$$

Solution Approach – Single-Level Reformulation

- Now, we formulate the KKT conditions to provide both necessary and sufficient conditions for the optimality of UE link flows in (a) (Farvaresh and Sepehri, 2011):

$$0 \leq f_a^{p,od} + f_a^{t,od} \perp \left[C_a - \left(\lambda_u^{od} - \lambda_v^{od}\right)\right] \geq 0, \forall a = (u,v) \in A,\ od \in OD,$$

(k)-(n) and (p).

which is equivalent to

$$f_a^{p,od} + f_a^{t,od} \leq \overline{F}\,\xi_a^{od},\ \forall a \in A,\ od \in OD \quad \text{(t1)}$$

$$C_a - \left(\lambda_u^{od} - \lambda_v^{od}\right) \leq \overline{\Lambda}\,(1 - \xi_a^{od}),\ \forall a \in A,\ od \in OD \quad \text{(t2)}$$

$$C_a - \left(\lambda_u^{od} - \lambda_v^{od}\right) \geq 0,\ \forall a \in A,\ od \in OD \quad \text{(t3)}$$

$$\xi_a^{od} \in \{0,1\},\ \forall a \in A,\ od \in OD \quad \text{(t4)}$$

(k)-(n) and (p),

where $\overline{F}$ and $\overline{\Lambda}$ are the upper bounds of $f_a^{p,od} + f_a^{t,od}$ and $C_a - \left(\lambda_u^{od} - \lambda_v^{od}\right)$

Solution Approach – Single-Level Reformulation

- We ensure constraints (r3) are convex and binding by applying a piece-wise linear function (Wang and Lo, 2010; Farvaresh and Sepehri, 2011) to approximate the user cost function, and adding binary variables $\sigma_{n',a}$ accordingly

$$C_a \geq C_a(x_a,\, z_a^p,\, z_a^t),\ \forall a \in A \Rightarrow$$

$$C_a \geq L_{n',a}(x_a, z_a^p, z_a^t),\ \forall n' \in \{1, 2, 3, \ldots, N'\}, \forall a \in A, \quad \text{(s1)}$$

$$C_a \leq L_{n',a}(x_a, z_a^p, z_a^t) + \overline{C}_a(1 - \sigma_{n',a}),\ \forall n' \in \{1, 2, 3, \ldots, N'\}, \forall a \in A, \quad \text{(s2)}$$

$$\sum_{n' \in N} \sigma_{n',a} = 1,\ \forall a \in A, \quad \text{(s3)}$$

$$\sigma_{n',a} \in \{0, 1\},\ \forall a \in A, \quad \text{(s4)}$$

where

$L_{n',a}(x_a, z_a^p, z_a^t)$; the tangent plane of the user cost function at n'^{th} point

$\overline{C}_a$ is the upper bound of the cost on link $a \in A$

$$L_{n',a}(x_a + z_a^t, z_a^p) = C_a(\zeta_1^{n'}, \zeta_2^{n'}) + \left[\frac{\partial C_a}{\partial(x_a + z_a^t)}(\zeta_1^{n'}, \zeta_2^{n'})\right](x_a + z_a^t - \zeta_1^{n'}) + \left[\frac{\partial C_a}{\partial(z_a^p)}(\zeta_1^{n'}, \zeta_2^{n'})\right](z_a^p - \zeta_2^{n'})$$

Solution Approach – Single-Level Reformulation

$$\underset{\mathbf{y}\in\{0,1\}^{|J|},\,\mathbf{x},\mathbf{z}^p,\mathbf{z}^t,\boldsymbol{\lambda},\mathbf{f},\mathbf{q}\geq 0}{\text{minimize}} \quad \sum_{j\in J}\eta_j y_j + \sum_{a\in A} M_a(x_a, z_a^p, z_a^t) + \sum_{a\in A}(x_a + z_a^p + z_a^t)\, C_a(x_a, z_a^p, z_a^t)$$

subject to (b)-(i), and

$$\lambda_u^{od} - \lambda_v^{od} \leq C_a,\ \forall a = (u,v) \in A,$$

$$C_a \geq L_{n',a}(x_a, z_a^p, z_a^t),\ \forall n' \in \{1,2,3,\ldots,N'\}, \forall a \in A,$$

$$C_a \leq L_{n',a}(x_a, z_a^p, z_a^t) + \overline{C}_a(1-\sigma_{n',a}),\ \forall n' \in \{1,2,3,\ldots,N'\}, \forall a \in A,$$

$$\sum_{n'\in N}\sigma_{n',a} = 1,\ \forall a \in A,$$

$$\sigma_{n',a} \in \{0,1\},\ \forall a \in A,$$

$$f_a^{p,od} + f_a^{t,od} \leq \overline{F}\,\xi_a^{od},\ \forall a \in A,\ od \in OD$$

$$C_a - \left(\lambda_u^{od} - \lambda_v^{od}\right) \leq \overline{\Lambda}\,(1-\xi_a^{od}),\ \forall a \in A,\ od \in OD$$

$$C_a - \left(\lambda_u^{od} - \lambda_v^{od}\right) \geq 0,\ \forall a \in A,\ od \in OD$$

$$\xi_a^{od} \in \{0,1\},\ \forall a \in A,\ od \in OD$$

(k)-(n) and (p).

Solution Approach – Single-Level Reformulation

Single level NLP reformulation

Formulate KKT conditions to provide both necessary and sufficient conditions for the optimality of UE link flows in (1)

Ensure constraints (3) are convex and binding by approximating the user cost function using a piece-wise linear function

(1) $$\underset{\mathbf{y}\in\{0,1\}^{|J|},\mathbf{x},\mathbf{z}^p,\mathbf{z}^t,\mathbf{b},\mathbf{f},\mathbf{q}\geqslant 0}{\text{minimize}} \quad \sum_{j\in J}\eta_j y_j + \sum_{a\in A} M_a(x_a, z_a^p, z_a^t) + \sum_{a\in A}(x_a + z_a^p + z_a^t)C_a(x_a, z_a^p, z_a^t)$$

subject to (b)-(i), (k)-(n), (p), and

(2) $$\lambda_u^{od} - \lambda_v^{od} \leqslant C_a, \quad \forall a = (u, v) \in A,$$

(3) $$C_a = C_a(x_a, z_a^p, z_a^t), \quad \forall a \in A.$$

$$0 \leqslant f_a^{p,od} + f_a^{t,od} \perp \left[C_a - \left(\lambda_u^{od} - \lambda_v^{od}\right)\right] \geqslant 0, \quad \forall a = (u, v) \in A, \ od \in OD,$$

$$\Downarrow$$

$$f_a^{p,od} + f_a^{t,od} \leqslant \overline{F}_a^{od}\ \xi_a^{od}, \quad \forall a \in A, \ od \in OD,$$

$$C_a - \left(\lambda_u^{od} - \lambda_v^{od}\right) \leqslant \overline{\Lambda}_a^{od}\left(1 - \xi_a^{od}\right), \quad \forall a \in A, \ od \in OD,$$

$$C_a - \left(\lambda_u^{od} - \lambda_v^{od}\right) \geqslant 0, \quad \forall a \in A, \ od \in OD,$$

$$\xi_a^{od} \in \{0, 1\}, \quad \forall a \in A, \ od \in OD,$$

$$C_a \geqslant L_{n',a}(x_a, z_a^p, z_a^t), \quad \forall n' \in \{1, 2, 3, \ldots, N'\}, \ a \in A,$$

$$C_a \leqslant L_{n',a}(x_a, z_a^p, z_a^t) + \overline{C}_a(1 - \sigma_{n',a}), \quad \forall n' \in \{1, 2, 3, \ldots, N'\}, \ a \in A,$$

$$\sum_{n'\in N}\sigma_{n',a} = 1, \quad \forall a \in A,$$

$$\sigma_{n',a} \in \{0, 1\}, \quad \forall n' \in \{1, 2, 3, \ldots, N'\}, \ a \in A.$$

$$L_{n',a}(x_a + z_a^t, z_a^p) = C_a\left(\zeta_1^{n'}, \zeta_2^{n'}\right) + \left[\frac{\partial C_a}{\partial(x_a + z_a^t)}\left(\zeta_1^{n'}, \zeta_2^{n'}\right)\right]\left(x_a + z_a^t - \zeta_1^{n'}\right) + \left[\frac{\partial C_a}{\partial(z_a^p)}\left(\zeta_1^{n'}, \zeta_2^{n'}\right)\right]\left(z_a^p - \zeta_2^{n'}\right).$$

Solution Approach

Proposition 2. *If $g_1 > 0.5$, $e_0 \gg e_1$, the objective function (1) is convex when every link flow falls into the following domains:*

$$(x + z^t) \geq \max_{\Gamma \in [\Gamma_{min}, \Gamma_{max}]} \left(\frac{-\Omega_2(\Gamma) - \sqrt{\Omega_2^2(\Gamma) - 4\Omega_1(\Gamma)\Omega_3}}{2\Omega_1(\Gamma)} \right), \text{ and}$$

$$(x + z^t) \leq \min_{\Gamma \in [\Gamma_{min}, \Gamma_{max}]} \left(\frac{-\Omega_2(\Gamma) + \sqrt{\Omega_2^2(\Gamma) - 4\Omega_1(\Gamma)\Omega_3}}{2\Omega_1(\Gamma)} \right), \text{ and}$$

$$(x + z^t) \geq \max_{\Gamma \in [\Gamma_{min}, \Gamma_{max}]} \left\{ \frac{\pi_4(\pi_1\pi_6 - \pi_3\pi_4) + \pi_5(\pi_2\pi_6 - \pi_3\pi_5)}{-(\pi_4\Gamma + \pi_5)(\pi_1\pi_5 - \pi_2\pi_4)} \right\}, \text{ if } \delta > 0, \text{ or}$$

$$(x + z^t) \leq \max_{\Gamma \in [\Gamma_{min}, \Gamma_{max}]} \left\{ \frac{\pi_4(\pi_1\pi_6 - \pi_3\pi_4) + \pi_5(\pi_2\pi_6 - \pi_3\pi_5)}{-(\pi_4\Gamma + \pi_5)(\pi_1\pi_5 - \pi_2\pi_4)} \right\}, \text{ if } \delta < 0,$$

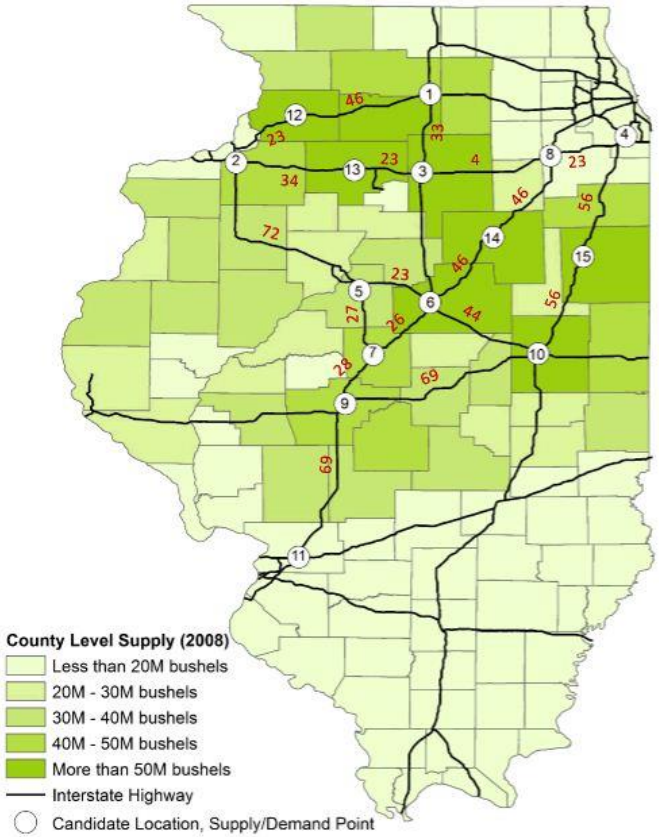
where $\Omega_1(\Gamma) = -(\pi_1\pi_5 - \pi_2\pi_4)^2 \left(4\pi_4\pi_5\Gamma + (\pi_4 - \pi_5\Gamma)^2\right)$,
$\Omega_2(\Gamma) = (\pi_1\pi_5 - \pi_2\pi_4)\left(4\pi_4\pi_5\Gamma(\pi_2\pi_6 - \pi_3\pi_5) - 2(\pi_4 - \pi_5\Gamma)\left(2\pi_3\pi_4\pi_5 - \pi_6(\pi_1\pi_5 - \pi_2\pi_4)\right) - \pi_1\pi_6 + \pi_3\pi_4\right)$,
$\Omega_3 = 4\pi_4\pi_5(\pi_1\pi_6 - \pi_3\pi_4)(\pi_2\pi_6 - \pi_3\pi_5) - (2\pi_3\pi_4\pi_5 - \pi_6(\pi_1\pi_5 - \pi_2\pi_4))^2$, $\pi_1 = lm_1rg_3 \ln^{-1}(\frac{g_1}{1-g_1})(g_2m_1e_0 + c_1(2g_1 - 1))$, $\pi_2 = lm_1rg_3 \ln^{-1}(\frac{g_1}{1-g_1})(g_2m_1e_1 + c_1(2g_1 - 1))$, $\pi_3 = lm_1^2g_2g_3re_2 \ln^{-1}(\frac{g_1}{1-g_1})$, $\pi_4 = c_1g_1 + e_0m_1g_2$, $\pi_5 = c_1g_1 + e_1m_1g_2$, $\pi_6 = m_1g_2(e_2 - r)$, *and* $\delta = -(\pi_4\Gamma + \pi_5)(\pi_1\pi_5 - \pi_2\pi_4)$,

- For a typical problem setting, the problem is convex over $0 \leq x + z^t \leq 5 \times 10^4 \ (pc/hr)$.

Case Study

Illinois Biofuel Development

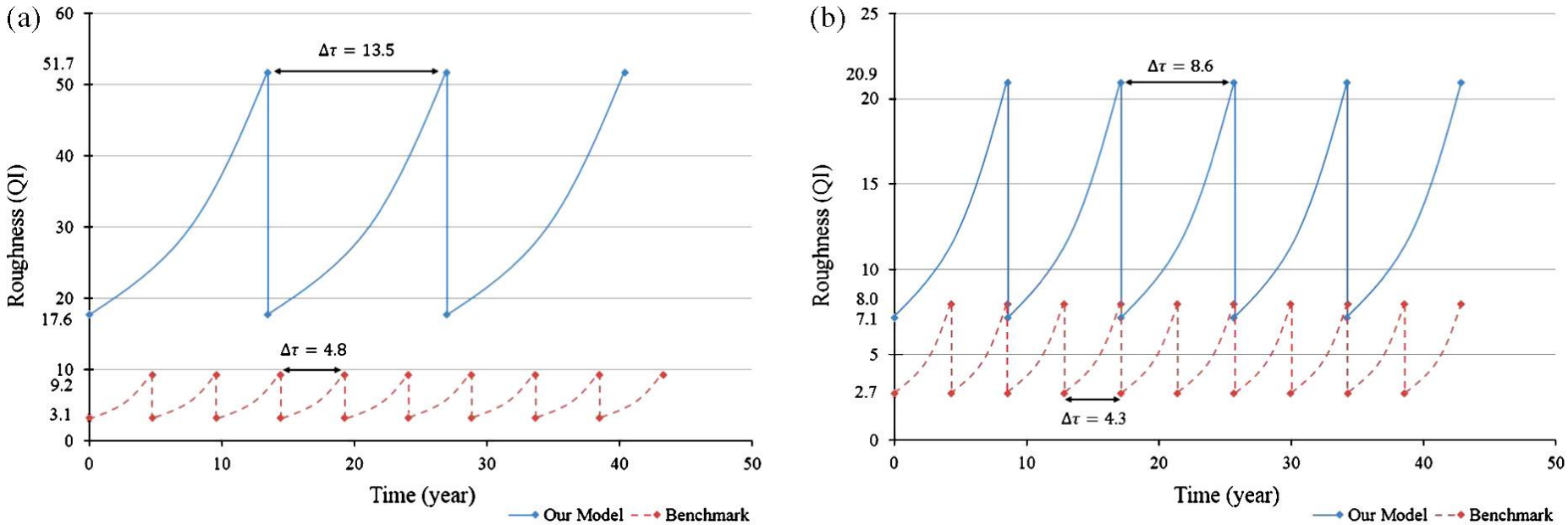
- Network has 17 nodes, 42 links, 34 OD pairs
- Candidate refinery locations, $J=\{1,2,\ldots,10\}$
- Biomass supply locations, $I=\{6,9,10,\ldots,17\}$
- Biofuel demand locations, $K=\{4,5,7,8,11\}$



Scenario	Facility location	Facility cost ($\times 10^9$ \$)	Agency cost ($\times 10^9$ \$)	Delay cost ($\times 10^9$ \$)	Veh. cost ($\times 10^9$ \$)	Total user cost ($\times 10^9$ \$)	Shipment cost($\times 10^9$ \$)	Total cost ($\times 10^9$ \$)
Benchmark	1, 2, 6	0.081	0.081	5.345	0.233	5.578	0.280	5.741
Our model	3, 9, 10	0.081	0.060	5.058	0.140	5.198	0.079	5.339
Difference	–	–	25.9%	5.4%	39.9%	7.3%	71.8%	7.0%

* Background OD traffic flow and road network: RITA, 2008; IDOT, 2007

Numerical Results: Compare to Benchmark

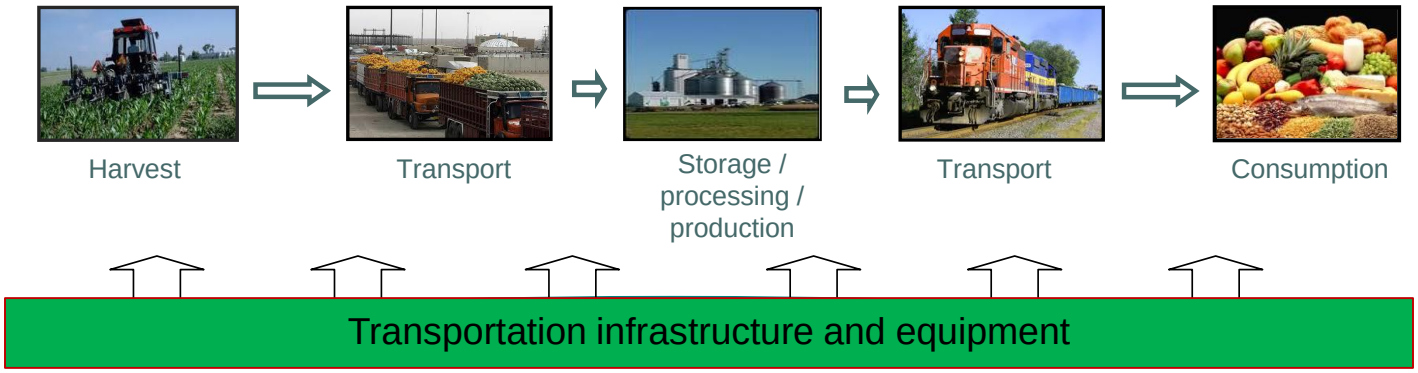


Optimal roughness trajectory in Illinois network; (a) pavement segment 10 → 6 and (b) pavement segment 9 → 10

Related Publications

- Bai, Y., Hernandez-Urrea, J.A., Gungor, O.E., Ouyang, Y. and Al-Qadi, I.L. (2015) "Optimal pavement design and rehabilitation planning using a mechanistic-empirical approach." EURO Journal on Transportation and Logistics, 4(1): 57-73.
- Hajibabai, L., Bai, Y. and Ouyang, Y. (2014) “Joint optimization of freight facility location and pavement infrastructure rehabilitation under network traffic equilibrium.” Transportation Research Part B, 63: 38-52.
- Ouyang, Y. and Madanat, S. (2006) “An analytical solution for the finite-horizon pavement resurfacing planning problem." Transportation Research Part B, 40(9): 767-778.
- Ouyang, Y. and Madanat, S. (2004) “Optimal scheduling of rehabilitation activities for multiple pavement facilities: Exact and approximate solutions.” Transportation Research Part A, 38(5): 347-365.

Today's Topics



1. Agricultural supply chain design considering post-harvest loss and shipment congestion
2. Agricultural land use competition and government regulations
3. Joint planning of agricultural supply chain and roadway infrastructure rehabilitation

Thank you! / Obrigado!

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